

MODEL OF DEVELOPMENT OF INHOMOGENEITIES INSIDE THE IONOSPHERE *

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ABSTRACT

The propagation of signals across the ionosphere is affected in two ways. The average variation in the index of the medium creates propagation errors: distance, angle, Doppler, etc. Fluctuations in electronic density in the medium, and consequently in the index, create scintillations. The latter are related to the characteristics of the inhomogeneities in the medium. In the following article we will set out a model representative of these inhomogeneities allowing calculation of the scintillations of transmitted signals.

1. INTRODUCTION

Inhomogeneities in the ionosphere are essentially a nighttime phenomenon, occurring at equatorial ($-20^\circ < 20^\circ$) and polar ($> 55^\circ$) latitudes, and within the F layer (300 km $<$ 500 km). The creation of these inhomogeneities is the result of different mechanisms. The resulting production of irregularities is caused by:

- the amplification of local instabilities in the F region;
- fluctuations in the electric field;
- hydromagnetic waves created above the ionosphere which propagate up to the F region;
- highly energetic particles located in the upper part of the F region.

These irregularities develop under the effect of instability mechanisms (ExB gradient drift, streaming instabilities (Kelvin Helmholtz), Rayleigh Taylor, etc.). Characteristic dimensions and different growth rates correspond to each of the existing processes. The overall problem is very complex, therefore we have restricted the following analysis to the most representative case in order to highlight the key elements of the problem.

The first part of this article describes two approaches used to model the inhomogeneities. The initial approach is based on the development of a physical model. The electronic density in the medium and its fluctuations are

calculated by solving the medium's fluid equations. The second approach is focused on applications and relies on a statistical analysis.

The physical model is composed of two sub-models corresponding to developments to the first and the second order. The first order model provides the average value of the electronic density in the ionosphere between 100 and 1500 km altitude. The second order model is based on the same principle but here a more precise description of the physical phenomena has been introduced in order to highlight the development of inhomogeneities in the medium.

An analysis of the performances of the two physical models is then carried out, especially with regard to the scale of the inhomogeneities' dimensions required for telecommunication applications.

The second part of this article presents an applications-oriented model. It is based on a synthesis of inhomogeneities using the statistical characteristics of the latter. The related CPU time is minimal. This model can be used to carry out parametric studies and to rapidly assess the influence of inhomogeneities on the signal propagation. In particular, it can be used in a propagation code to calculate the amplitude of scintillations in the transmitted signal, and to deduce the margins to be included in calculation of the budget link for a given confidence level. An example of this calculation is presented at the end of the paper.

2. PHYSICAL MODEL

2.1. First order model

As a first approximation, within the range of altitude in question, there exists three species in the medium: O, N₂ and O₂, with O as the major constituent. Their respective concentrations can be obtained by the MSIS code. The numerical resolution applies to the calculation of the density of the O⁺ ion. The electronic density is assumed to be equal to this density, as the medium is electrically neutral. The other two constituents are nevertheless considered in the calculation of electron ion recombinations.

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The O^+ density is obtained by solving the medium's fluid equations: continuity, momentum and energy. The equations are as follows:

$$\frac{d N_i}{d t} = P_i - L_i - N_i \operatorname{div} v_d - B \frac{\partial}{\partial s} \left(\frac{N_i v_i}{B} \right)$$

where P_i and L_i are the production and loss rates for O^+
 v_d is the drift velocity

v_i is the ion velocity

B is the earth's magnetic field.

Momentum equation

$$v_i - v_n = - \frac{1}{m_i v_{in}}$$

$$\left[\frac{k T_i}{N_i} \nabla N_i + \frac{k T_e}{N_e} \nabla N_e + k \nabla (T_i + T_e) - m_i g \right]$$

where T_i and T_e are the electronic and ionic temperatures and v_n is the neutral wind velocity.

Solving these equations provides us with a parabolic equation with respect to time.

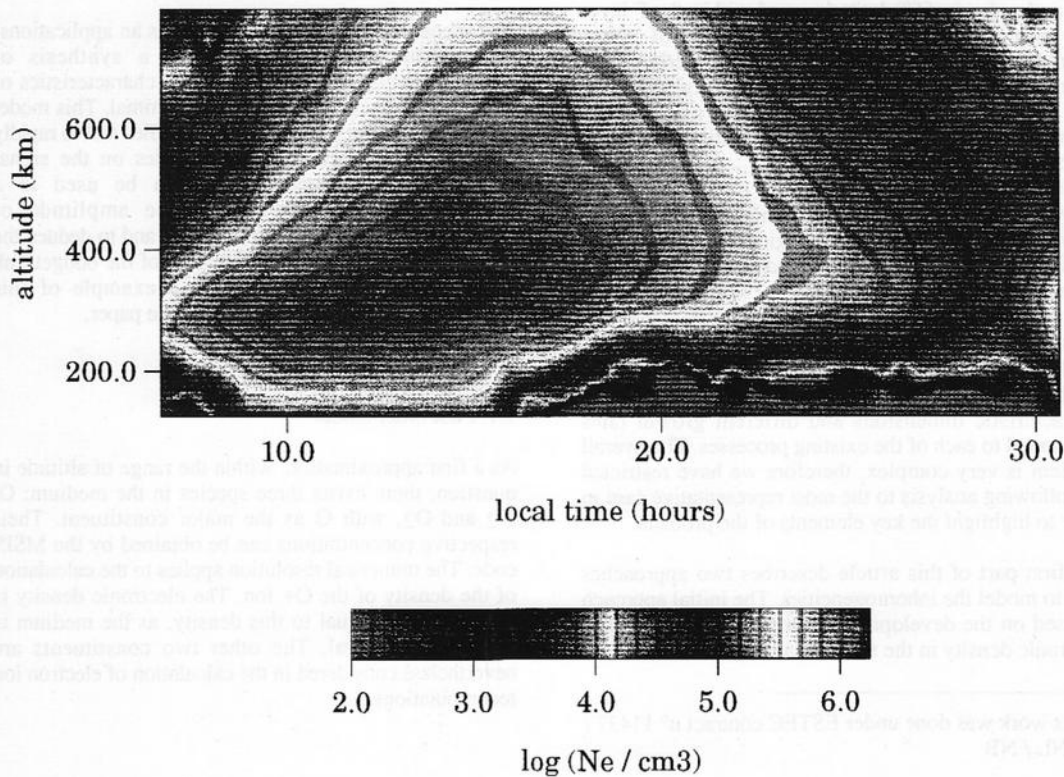
Algorithm

A succession of meridian planes are taken into consideration. The difference in longitude between these planes must remain small compared to the time scale. Among the simulation results presented hereafter, the difference in longitude is equal to 2° , which corresponds to a time step of 480 s due to the earth's rotation. The reference axes in each meridian plane include the earth's radius at the equator level, and the earth's magnetic field. The meshing in the meridian plane forms symmetrical curves with respect to the equator. The corresponding altitude of each point along these curves is maximum at the equator and decreases with latitude. Consequently, the number of calculations can be lowered if we limit the range of latitudes taken into account.

Input includes:

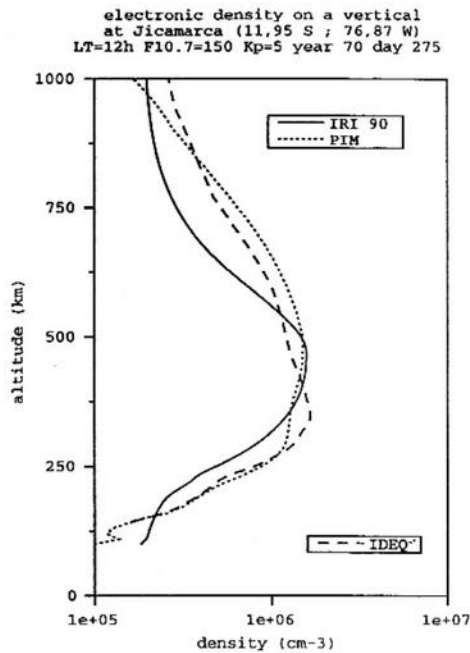
- densities of O , N_2 and O_2 provided by MSIS 90;
- neutral temperatures also provided by MSIS 90;
- production and loss rates of electrons;
- neutral velocity;
- vertical drift velocity.

These data have been selected from average profiles. Results could be more accurate if we could get real-time data. Temperature profiles for ions and electrons can be calculated by solving the energy equation. However, the calculation can be made easier by using typically temperature profiles and skipping the energy equation.



The parabolic equation obtained is solved by the Crank-Nicholson fully implicit scheme. At the bottom of the mesh, the medium is assumed to be in photochemical equilibrium. This sets the boundary conditions at the lowest altitude. The matrix of the system is tridiagonal, the solution is obtained by separating it into two triangular matrices.

An example of the result obtained is presented in figure 1. This figure shows the variation in electronic density, as calculated by the IDEQ code (Ionospheric Density at Equatorial regions) developed by IEEA, versus local time in Jicamarca (Peru), which is located very near the magnetic equator. A comparison of this result with those provided by other sources is presented in figure 2. The vertical profile at 12 noon local time is compared with those provided by the IRI 90 (International Reference Ionosphere) and PIM (Parameterized Ionospheric Model) codes. The space step of the vertical mesh used in this example is 10 km.



As compared to measurements at Jicamarca, the PIM code seems to provide more accurate results. The IDEQ results are very similar to PIM's at low altitudes. At the F layer altitude and above some discrepancies appear which can probably be explained by the differences in the input data set.

2.2. Second order model

Drawing up a model to predict electronic density to the second order relies on solving the medium's fluid

equations as described earlier. This second order model only takes momentum and continuity equations into account.

Continuity equation

The continuity equation can be written as such:

$$\frac{\partial N_i}{\partial t} + \nabla \cdot (N_i v_i) = P_i - L_i$$

In what follows, we assume that the production and loss rates of electrons are set at zero. The first hypothesis corresponds to nighttime and the second to high altitudes, typically the F layer and above.

Momentum equation

The momentum equation is as follows:

$$v_i - v_n = \frac{\Omega_i}{v_i} \left(\frac{E}{B} - v_i \times \hat{z} \right) + \frac{1}{v_i} \left(\frac{\partial}{\partial t} + v_i \cdot \nabla \right)$$

To solve the equations, the z-axis is aligned with the earth's magnetic field. Moreover, we assume the terms related to the gradients in electron and ion temperatures are set at zero. The forces applied, which correspond to the right hand side of the equation, are linked to the local electromagnetic field. The two terms used in the preceding equation apply to the ExB gradient drift instability and to the Kelvin-Helmholtz instability corresponding to magnetosphere-ionosphere coupling.

Methods

The preceding equations are transformed by using the previously indicated hypotheses, and assume that the electric field is of electrostatic nature. The equations are written as a differential equation versus time, and an elliptic equation.

The continuity equation can be written as:

$$\frac{\partial N}{\partial t} = \frac{1}{B} \left[\frac{\partial \Phi}{\partial y} \frac{\partial N}{\partial x} - \frac{\partial \Phi}{\partial x} \frac{\partial N}{\partial y} \right]$$

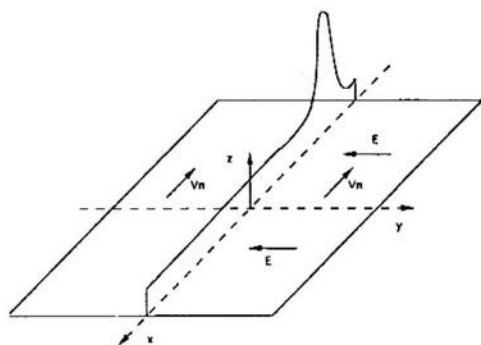
and the momentum equation as:

$$\nabla \cdot N \nabla \Phi = - B v_n \frac{\partial N}{\partial y}$$

The local electric field is assumed as being aligned with the x-axis. The medium is electrically neutral. The terms linked to Kelvin-Helmholtz instability have been omitted from the above equation.

Algorithm

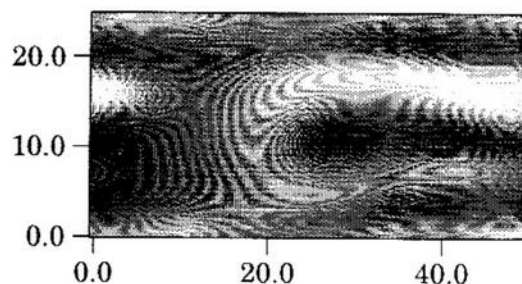
The analyzed area is rectangular. Periodical boundary conditions are assigned to one of the coordinates, and a Neumann boundary condition to the other coordinate. When starting the calculation, the electronic density is imposed on a specific point. It is assumed that the electronic impulse is gaussian in the example presented.



The continuity and momentum equations are successively resolved for each time step. The momentum equation provides the scalar potential for every point in the mesh. The values obtained are then used in the continuity equation to calculate the electronic densities at the following time step.

In the simulations, the space step was set at 1 km on one of the axes, and at 250 m on the other. This results in solving large-size problems, which typically involve several thousand points for analyzed areas representing tens of km on each of the axes. This is done by using the Incomplete Cholesky Conjugate Gradient technique (ICCG).

An example of result obtained after 200 s. is presented below. Dimensions of the analyzed area are 25 km x 50 km. The medium appears to be striated and these striations developed along the direction of vector $E \times B$. Increasing the analysis time leads to a breaking of these structures into smaller ones.



2.3 Global model

The time and space scales used for the two types of models, first order and second order, are very different, thereby making it very difficult to consider a simultaneous solution. However, if it is shown that small scale structures do not influence the large scale evolution, the shock capturing technique just described in section 2.2 may be appropriate in a global approach by refining locally the mesh both in terms of time and space.

3. NUMERICAL MODEL

The measurements available make it possible to estimate the probability of occurrence of inhomogeneities as a function of latitude, altitude and time, as well as their dimensions. In addition, the spectral density of fluctuations in electronic density can be approximated by a power law, whose slope is between -2 and -4. The lower cut-off frequency corresponds to the mean value of the size of the inhomogeneities L_0 . The related function is:

$$\gamma_{\Phi}(q) = \frac{C_p}{(q_0^2 + q^2)^{p/2}}$$

with $q_0 = 2\pi / L_0$

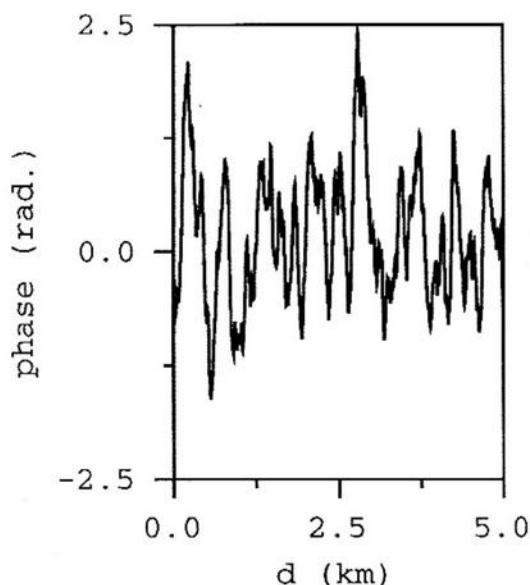
The autocorrelation function obtained by inverse Fourier transform can be calculated analytically. For propagation problems through the medium, we look at phase fluctuations in the transmitted signal in a plane perpendicular to the propagation direction. These phase fluctuations are directly related to electronic density through:

$$\sigma_{\Phi}^2 = C_s (\lambda r_e)^2 L L_0 \sigma_{N_e}^2$$

where L is the thickness of the medium,
and C_s is the medium's structure constant.

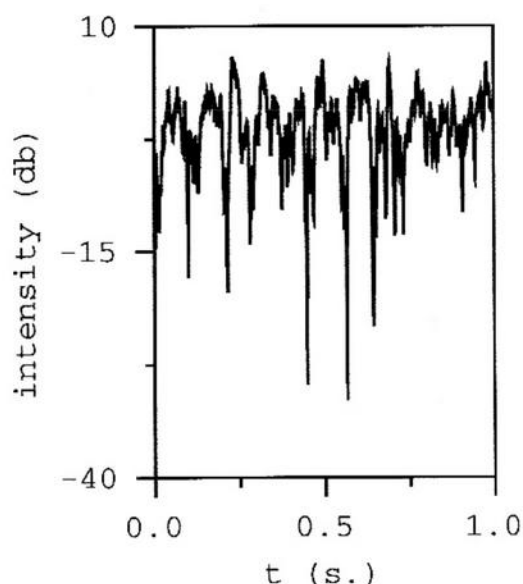
Moreover, the signal's phase is a centered gaussian random variable.

The electronic density fluctuations in the medium are synthesized using a technique borrowed from numerical filtering methods. The spectral density of the phase at the output of the medium is equal to the product of the Fourier transform of centered gaussian random variable and the square root of the spectral density of the signal we want to synthesize. The resulting random variable meets the required conditions. The corresponding signal is equal to the inverse Fourier transform of this product. The following figure presents results obtained with a slope of -3, a mean inhomogeneity size of 500 m and a phase standard deviation equal to 0.76 radians



Propagation in the medium can be calculated using the phase screen technique, by alternating scattering and propagation calculations. The related equation is the parabolic equation, and the corresponding technique applies regardless of the level of ionization within a screen. It applies in particular to the propagation outside the inhomogeneities layer.

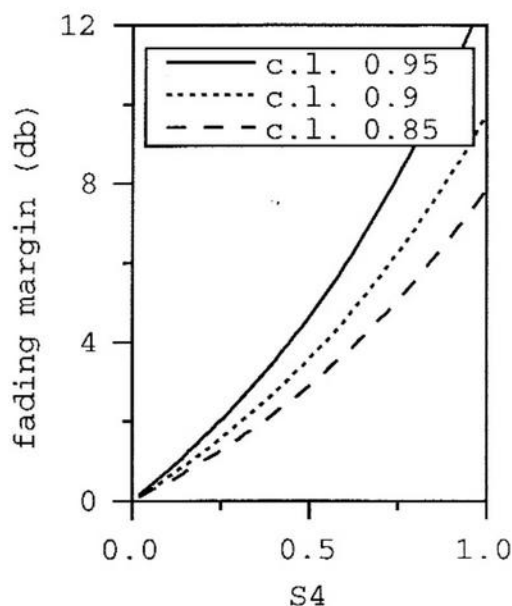
An example of a transmitted signal after propagation through an inhomogeneities layer 50 km thick, in the F region is presented below.



Fluctuations in the signal's intensity follow the m-Nakagami probability law, where parameter m of this law is equal to the inverse of the square of the scintillation ratio S4 defined as:

$$S_4^2 = \sigma_I^2 - 1$$

The variance of the intensity is calculated numerically using the results obtained from the intensity of the transmitted field. This makes it possible to define a margin to be included in calculation of the budget link for a given confidence level. The corresponding values are presented for confidence levels equal to 85, 90 and 95%.



The results presented in this paragraph assume an isotropic medium. The problem is 1D in a plane perpendicular to the propagation direction. This calculation is also possible by assuming an anisotropic medium. In this case, the spectral density of electronic density fluctuations in the medium is:

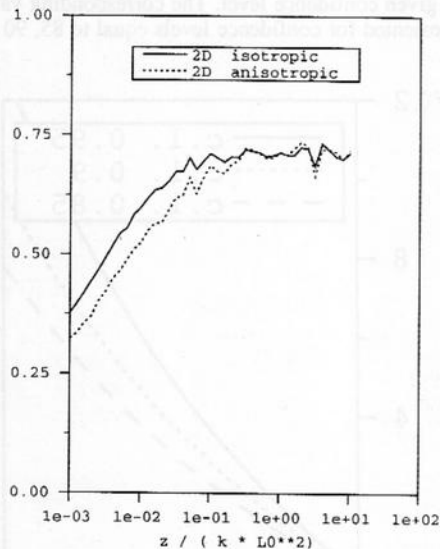
$$\gamma(q) = \frac{C_p}{(q_0^2 + A q_x^2 + B q_y^2)^{p/2}}$$

which includes two directions q_x and q_y and the ratio B/A represents the anisotropic ratio.

The numerical synthesis technique is identical. However, this time it introduces 2D Fourier transforms (see figure 8), and we compare the results obtained for 2 values of the B/A ratio. In the first case, this ratio is equal to 1 and the medium is isotropic. In the second case, the ratio is equal to 10.

The simulation results presented for the scintillation ratio help quantify the influence that the medium's anisotropy has on the scintillation ratio of the transmitted signal, from which we deduce the corresponding margin in the budget link. This result is presented as a function of the distance from the inhomogeneous medium to the receiver for average values of the phase standard deviation of the transmitted signal.

S4 value vs free space propagation distance
phase standard deviation = 0.58 r.



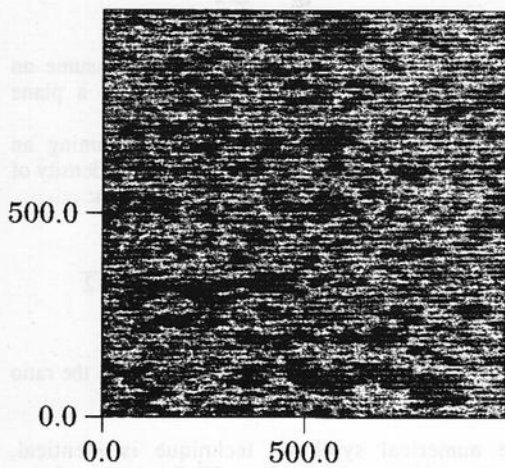
Generally speaking, the scintillation ratio increases with distance for low values up to an asymptotic value equal to 1 in the case of strong scintillations. The result of figure 8 shows the variation in the scintillation ratio linked to the medium's atmosphere. Both curves merge in the asymptotic regime, which is to be expected.

4. CONCLUSION

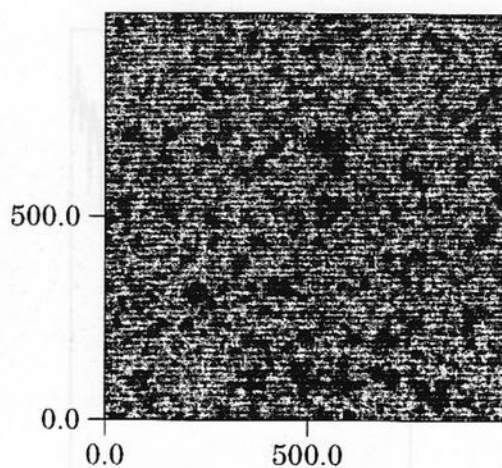
The model developed provides a few elements concerning the structure of the medium and the development of inhomogeneities. Broad-scale resolution of the problem is confronted with numerical difficulties due to the highly differing time and space steps of the first and second order models. The extent of the characteristics of the inhomogeneities may nevertheless be evaluated by studying small and large scale problems in a partially separated manner.

The results presented apply to the ExB gradient drift instability. There needs to be a systematic study of the different types of instability likely to develop in the medium, as this would provide statistical data on the characteristics of the inhomogeneities at different latitudes.

The numerical approach presented in paragraph 3 is, on the other hand, very well adapted to telecommunication problems, and provides faithful results as to the effects observed, that is, as long as precise statistical characteristics of the medium are known.



anisotropy ratio = 10



anisotropy ratio = 1