# THE RECIPROCITY THEOREM APPLIED TO FIND THE BEST COUPLING INCIDENCE

# RECHERCHE DE LA DIRECTION D'INCIDENCE DE PLUS FORT COUPLAGE A L'AIDE DU THEOREME DE RECIPROCITE


**Jean-Pierre ADAM[1], Jean-Christophe JOLY[2], Bernard PECQUEUX[2], Didier ASFAUX[1]**

[1]IEEA, 13 promenade Paul Doumer, 92400 Courbevoie, France
[2]Centre d'Etudes de Gramat, DGA/DET/CEG, 46500 Gramat, France

jean-pierre.adam@club-internet.fr



**Abstract**
This document presents an application of the electromagnetism reciprocity theorem in the time domain. The far field radiated by a system in a given direction is used to evaluate the effect of an incident plane wave coming from that direction. The useful formulas are derived for a current source and a voltage source. These results are applied with the FDTD software Gorf3D. A single run with a system radiating in N directions enables to predict the result of N runs with a system illuminated by a plane wave. Application examples are used to estimate the computation time gain.



**Key words** : Reciprocity theorem, time domain, FDTD



**Résumé**
Ce document présente une application du théorème de réciprocité en électromagnétisme dans le domaine temporel. La forme temporelle du champ rayonné par un système dans une direction est utilisée pour déterminer l'effet d'une onde plane provenant de cette direction. Les relations utiles sont montrées pour une source de courant et une source de tension. Ces résultats sont mis en œuvre à l'aide du code DFDT Gorf3D. Un seul cas de calcul détermine le champ rayonné dans N directions et permet de remplacer N cas de calcul avec une onde plane incidente. Des applications illustrent la méthode et permettent d'estimer le gain de temps de calcul.



**Mots clés** : Théorème de réciprocité, domaine temporel, DFDT


## 1) INTRODUCTION

The reciprocity theorem is a powerful simplification tool in electromagnetism. It is often applied in the frequency domain. On the contrary, the application of this principle in the time domain is not so common. This paper deals with this problem and illustrates it with applications using the FDTD (Finite Differences in Time Domain) Gorf3D code. In these examples, the parameter under study is the peak value of a wire current or load voltage when a system is illuminated by an incident plane wave. This state will be called "reception". In the second state called "transmission", the load is replaced by a generator. The reciprocity theorem is the link between both states. Here, it is used to determine the influence of the incidence direction and of the polarization of the plane wave.

## 2) TIME DOMAIN RECIPROCITY THEOREM

A full proof of a general time domain reciprocity theorem is not the aim of this work. A statement and proof of such a theorem may be found in [1] for example. The present paper is application oriented. However, it may be useful to recall some formulas and interpretations. The starting point is a consequence of the reciprocity theorem applied to an antenna in the frequency domain : the behavior of a receiving antenna only depends on the incident plane wave and the behavior of the antenna when it is transmitting. For the load, a receiving antenna can be represented as a Thevenin equivalent source. The impedance of this source is the same as the impedance of the antenna as a transmitter. When the antenna is illuminated by an incident electric field $\vec{E}_0$ coming from the direction $\vec{u}$, the open circuit voltage $V_{oc}$ of this source is given by (cf. [2]) :

$$V_{oc}(\omega) = \frac{4\pi\, c}{j\omega Z_0} \frac{\vec{F}(\omega).\vec{E}_0(\omega)}{I_t(\omega)} \qquad (1)$$

where $\vec{F}$ is deduced from :

$$\vec{E}_t(\omega) = \frac{\exp(-ikr)}{r} \vec{F}(\omega) \qquad (2)$$

$\vec{E}_t$ is the transmitted electric far field at a distance r in the direction $\vec{u}$, when the antenna is fed by a current source $I_t$. $Z_0$ is the impedance of the surrounding medium.

A direct transposition of $V_{oc}$ expression in the time domain would imply convolutions. In order to avoid these CPU (Central Processing Unit) time consuming operations, the feeding current $I_t$ is chosen to simplify the expression. A simple choice consists in setting the same time dependency for the current $I_t$ as for the incident field $\vec{E}_0$. In that case the formula (1) is simplified as following :

$$V_{oc}(\omega) = \frac{4\pi\, c}{j\omega Z_0} \mathrm{r}\, \mathrm{E}_t(\omega) \qquad (3)$$

equivalent in the time domain to :

$$V_{oc}(t) = \frac{4\pi\, c}{Z_0} \mathrm{r} \int_0^{t} \mathrm{E}_t(\tau)\, d\tau \qquad (4)$$

The step between (3) and (4) is obvious, but it shows the major difference between the frequency domain and the time domain. In (3), for a given frequency, the magnitude of $V_{oc}(\omega)$ is proportional to the magnitude of $E_t(\omega)$. This is the familiar consequence of the reciprocity theorem in the frequency domain : the receiving pattern is the transmitting pattern. The formula (4) shows that they are not exactly the same in the time domain. Indeed, $V_{oc}(t)$ is proportional to the time integral of $E_t(t)$. This statement is called “derivative relation” for transmission and reception in [3].

For simplification purposes, there are omissions in (3) and (4). The phase is not taken into account because it simply adds a delay in the time domain. In addition, the scalar product is omitted because the components $E_{t\theta}$ and $E_{t\phi}$ (spherical coordinates) of the radiated field are separately treated. This is illustrated with an arbitrary linear polarization. The polarization angle is noted $\alpha$. Formulas (1) and (2) are then replaced by :

$$V_{oc}(\omega) = \frac{4\pi\, c}{j\omega Z_0} r(E_{t\theta}(\omega)\cos(\alpha) + E_{t\varphi}(\omega)\sin(\alpha)) \frac{E_0(\omega)}{I_t(\omega)} \qquad (5)$$

Another remark is that the relation could be even simpler if the current $I_t$ is the integrated form of the incident field $\vec{E}_0$. But such a current waveform is not always easily available in the considered software.

An analog result is obtained when the antenna is fed by a voltage source. In that case, the transmitted electric far field is related to the short circuit current $I_{sc}$ observed when the antenna is illuminated by a plane wave. This relationship is :

$$I_{sc}(t) = -\frac{4\pi\, c}{Z_0} \mathrm{r} \int_0^{t} \mathrm{E}_t(\tau)\, d\tau \qquad (6)$$

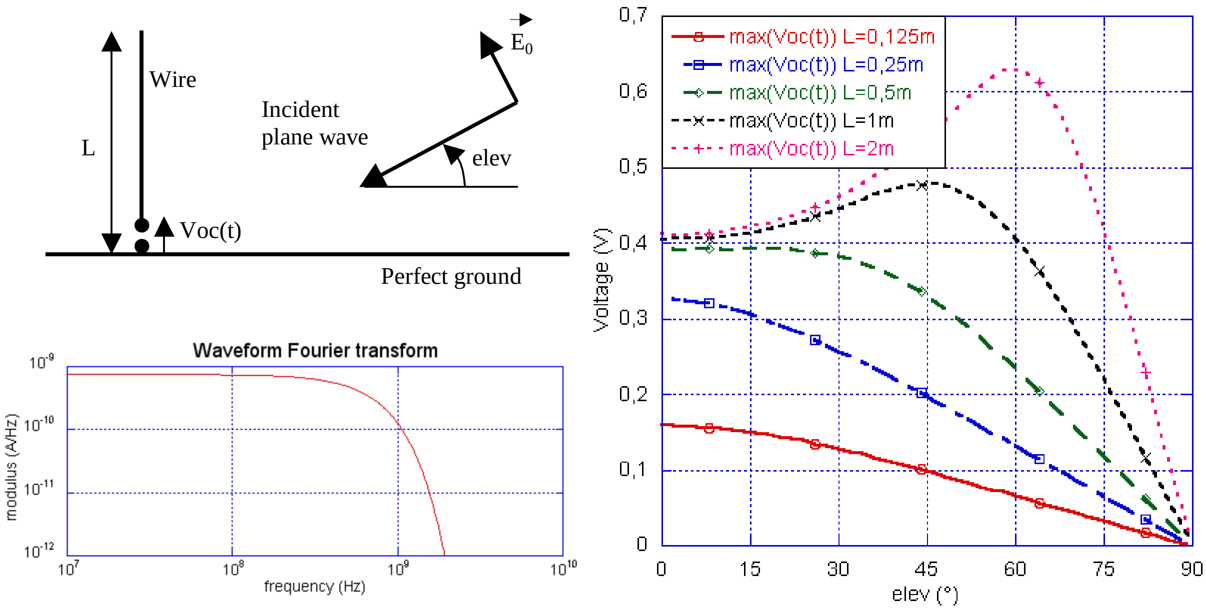
A serial impedance can be added beside the voltage generator in transmitting mode. In receiving mode, the generator is replaced by a short circuit and the current at this point is deduced from formula (6). If this point is near enough the serial impedance to meet the electrostatic condition, then the observed current is the same as the one flowing through the impedance. In other words, the method could easily be extended to compute a load current.

In addition, the voltage induced in a small dipole is proportional to the electric near field component which is parallel to the dipole. As a consequence, the reciprocity theorem may be applied to the far field radiated by this small dipole fed by a current source in order to evaluate the temporal variation of the near field at the location of the dipole when the system is illuminated by a plane wave.

## 3) APPLICATIONS

The time domain applications of the reciprocity theorem are usually found in the field of the UWB (Ultra-Wideband) antennas (cf. [3]). In this work, it is used for EMC (Electromagnetic Compatibility) applications : the influence of the incidence direction of an EMP (Electromagnetic Pulse) is studied. The simulations are performed with the FDTD software Gorf3D developed by the DGA (French Defense Administration). A description of the FDTD method may be found in [4] and some details of its software implementation are presented in [5]. The tool includes a near field to far field transformation routine, allowing consequently computing the field radiated outside the FDTD domain. This transformation is performed in the time domain.

The first example is a vertical monopole antenna placed over a perfect ground and fed by a current source. The reciprocity theorem is applied to compute the maximum value of the open circuit voltage that would be implied by an incident plane wave. This incident plane wave is vertically polarized. In this work, the maximum value of the voltage is studied. However, this is not a limitation : any other time domain treatment is applicable, like for example derivation, integration, filtering, FFT… Figure 1 presents the results obtained for different monopole lengths. This figure also shows the frequency spectrum of the pulse used to feed the antenna. This spectrum drops down after 300 MHz. This frequency corresponds to a wavelength λ of 1 m. For small monopoles (length below λ/4 = 0.25 m), the maximum value is achieved with an incident plane wave perpendicular to the wire. This result is classical in the frequency domain. For longer monopoles, the maximum coupling is reached with a non-zero elevation angle.




(a) (b)

*Figure 1*: (a) upper part : monopole antenna in receiving mode; lower part : frequency spectrum of the incident plane wave (receiving mode) and of the feeding current (transmitting mode). (b) maximum value of the open circuit voltage (deduced by reciprocity) as a function of incidence direction, for different monopole lengths.

The term “antenna” includes the radiating element and the whole carrier, as presented in figure 2. This shielded truck carries a vertical monopole antenna. The structure is placed over a perfect ground. Like in the previous example, the maximum value of the open circuit voltage induced at the monopole feed port when illuminated by a vertically polarized plane wave is analyzed. This polarization is chosen to maximize the coupling with the wire antenna. The waveform is presented in figure 2a. It corresponds to a typical EMP waveform. Again, the reciprocity theorem is used to reduce the case in only one simulation run in transmitting mode. The influence of the incidence direction has been investigated.

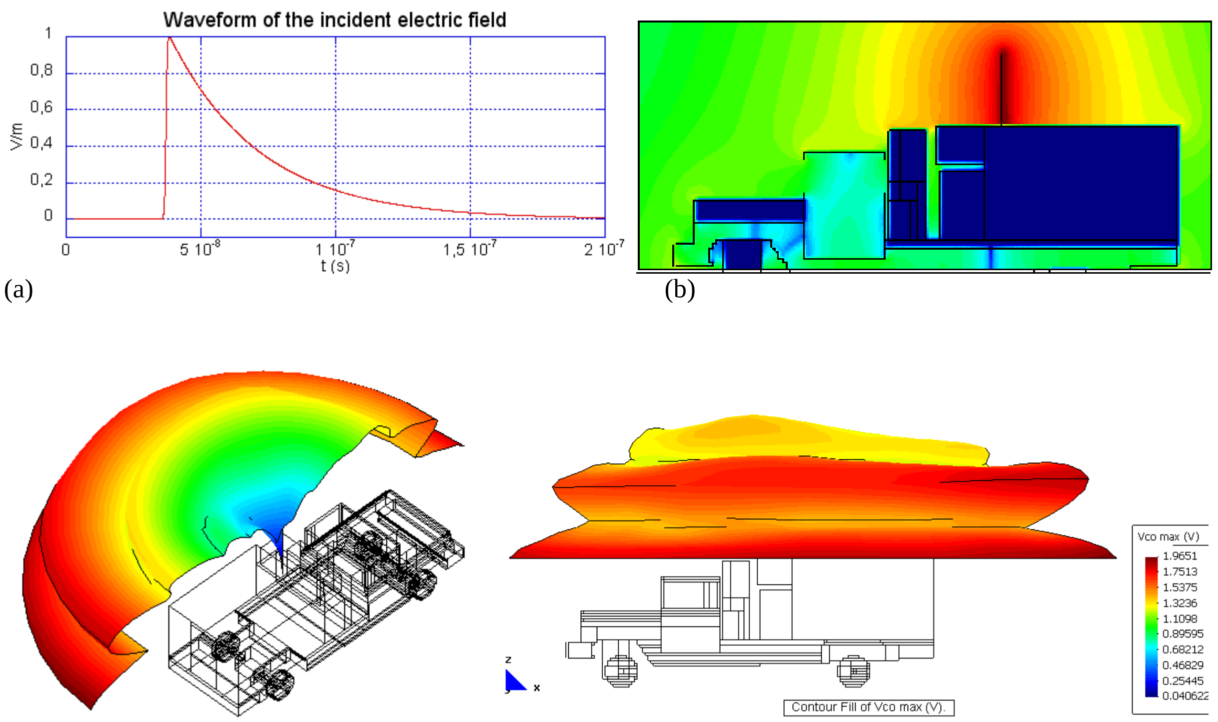
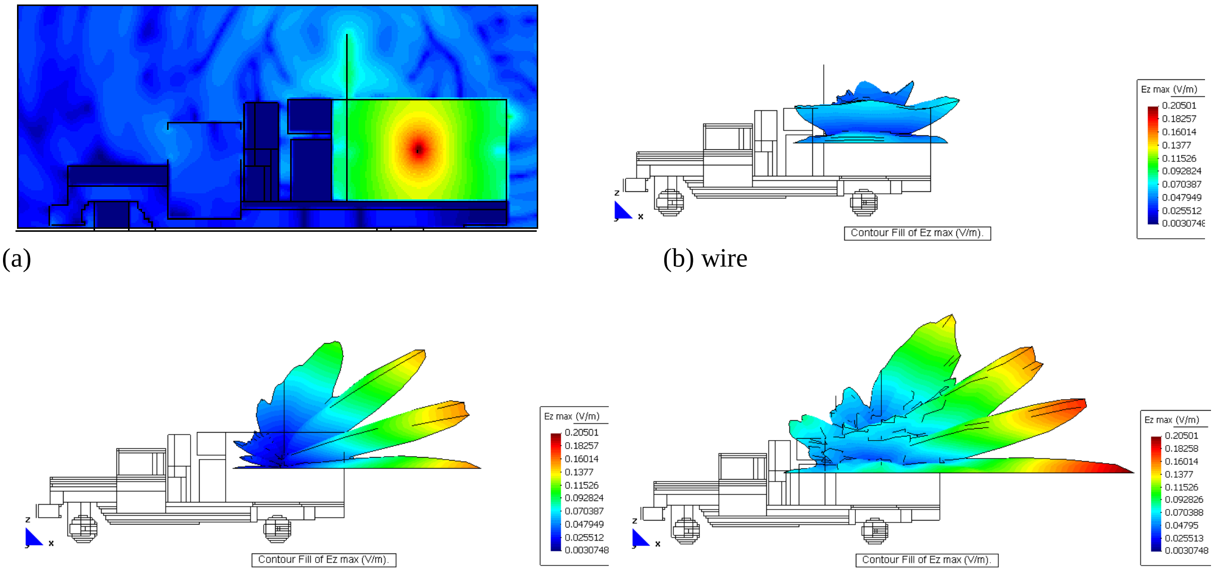



*Figure 2*: (a) waveform of the incident field (receiving mode) and of the feeding current (transmitting mode). (b) electric field (log scale) for a given time during the simulation of the monopole fed by a current source. (c) and (d) maximum value of the open circuit voltage induced in the antenna port as a function of the direction of incidence. (c) and (d) are deduced from the simulation shown in (b) with the use of the reciprocity theorem.

This complex example is a good case for estimating the CPU time reduction. The simulations were performed on a standard computer with an AMD Athlon64 3500+ processor. The FDTD domain contains 1.5 millions cells. A direct computation of the receiving mode requires 855 runs of 10 minutes, totaling 142 hours. The transmitting mode combined with the reciprocity requires 1 simulation of 16 hours with default configuration. The time consuming near to far field transformation is the cause of the CPU time increase. This computation time can be dropped to 30 minutes by tuning the under sampling options of the near to far field transformation. Despite this drawback, the use of the proposed method provides a significant speed up of 284 times. This observation is particularly true if the number of considered incidences is important.

The next example is based on the same structure and implies the same EMP. However, two other electromagnetic weak points are considered : a horizontal slot at the rear of the truck and a longer vertical wire going through the roof of the truck. The observed parameter is the maximum value of the electric field at the center of the truck when an EMP illuminates it. In order to apply the reciprocity theorem as it was presented previously, the observation point is replaced by a small dipole (a one FDTD cell size long wire). This vertical dipole is a sensor able to measure the vertical component of the electric field. The figure 3 presents the results for three different configurations : wire alone, slot alone, wire and slot. These different results may be used to investigate the privileged penetration way for a given direction of incidence. For example, the slot is the main coupling way for a wave coming from the rear of the truck. On the contrary, the coupling through the wire seems to be rather omnidirectional.




(a) (b) wire

(c) slot (d) wire and slot

*Figure 3*: (a) electric field (log scale) for a given time during the simulation of a small wire fed by a current source. There is a horizontal slot in the rear face. A vertical wire goes through the roof of the truck. (b), (c) and (d) maximum value of the vertical component of the electric field at the small wire location for different directions of incidence. (d) results are deduced by reciprocity from the computation shown in (a). (b) and (c) results are also deduced by reciprocity, but the slot and the wire are respectively absent.

## 4) CONCLUSION

The reciprocity theorem is applied in the time domain with a FDTD software. This method provides a fast evaluation of the best coupling direction of incidence. However, the results are available for only one observation point in the structure. This point is the source point in the transmitting mode. For example, the method is unable to predict a near field map. This seems to be the main limitation. This method is perfectly suited to study the influence of the incidence direction on a voltage or a current at a particular location, for example the input port of a vulnerable device.

## REFERENCES

[1] B. Ru-Shao Cheo, “A Reciprocity Theorem for Electromagnetic Fields with General Time Dependence”, *IEEE Trans. Antennas Propagat.*, vol. AP-13, pp. 278–284, March 1965.

[2] S.W. Lee, “Theorems and Formulas”, in *Antenna Handbook, Antenna fundamentals and mathematical techniques*, vol. 1, chap. 2, Edited by Lo and Lee, Chapman & Hall, 1993.

[3] Glenn S. Smith, “A Direct Derivation of a Single-Antenna Reciprocity Relation for the Time Domain”, *IEEE Trans. Antennas Propagat.*, vol. 52, no. 6, pp. 1568- 1577, June 2004.

[4] Allen Taflove, *Computational Electrodynamics, The Finite-Difference Time-Domain Method*, Artech House, 1995.

[5] L. Morel, B. Pecqueux, R. Vézinet, ”Notice d’installation et d’utilisation de la version exportable du code GORF3D”, Note interne CEG I94-28, October 1994.