Antenna radiation in the Presence of an Infinite Interface

Yannick Béniguel
IEEA
Courbevoie, France
beniguel@ieea.fr

Full Paper — **This paper addresses the problem of antennas analysis for radiating structures located at the vicinity of one or several infinite interfaces between different media. It is shown how to modify the field integral equations and how to implement the Sommerfeld integral calculations. Results obtained are presented on a typical bi conical HF antenna.**

Keywords-Mixed Potential integral equation, Sommerfeld integrals

I. INTRODUCTION

The electromagnetic field integral equation technique is a very well suited technique for antenna analysis. It allows considering surfaces and wire elements, including dielectric parts as lenses or substrates. Using the classical method of moments, the analysis technique only considers interfaces between media. These interfaces are meshed into surface or wire elements which constitute the problem unknowns. The case of infinite interfaces shall be considered modifying the Green's problem function to avoid the meshing on these parts. It is the object of this paper.

The VLF and HF antennas are examples of such antennas. They are placed on the ground. They usually include a ground plane constituted by radial wires placed on the interface. The electrical ground characteristics strongly influence the antenna radiating pattern and the VSWR. Another important example is the microstrip antennas. Analyzing such antennas using the method of moments solving the field integral equations implies considering both electric and magnetic currents on all interfaces. On such antennas the current is located close to the microstrip elements. It is consequently possible to consider the substrate interfaces of infinite extent. The developments presented in this paper can be applied under this hypothesis.

The radiation of a dipole element located at the vicinity of an interface was first derived by Sommerfeld [11] and has since been the object of a considerable number of papers.

The novelty of this paper is to present a recent technique allowing to tackle the problem of the calculation of the Sommerfeld integrals and its implementation in a numerical code. The corresponding developments have been implemented in the ICARE Antennas & RCS software [2], [7].

II. THE MIXED POTENTIAL INTEGRAL EQUATION

It has been shown [8], that there are three different ways to write the Lorentz gauge relationship in order to meet the boundary conditions on the interface. Although equivalent from a theoretical point of view, formulation C is the most convenient as regards its numerical implementation. This formulation has been used. With respect to the Electric Field Integral Equation (EFIE), the new equation, named Mixed Potential Integral Equation (MPIE), includes additional terms to take the Green's function modification into account.

In the case of a metallic structure, the MPIE equation is:

$$E^s = -j\,\omega\,A - \nabla\Phi \qquad (1)$$

With for the vector potential

$$A = \int \overline{G}^A(r,r') \, . \, J_s(r') \; ds' \qquad (2)$$

and $G^A = G^A_{vv} + \hat{z}\,\hat{u}\; G^A_{zu} + \hat{z}\,\hat{z}\; G^A_{zz}$

and for the scalar potential

$$\Phi = \frac{j\omega}{k_i^2} \int \left[\nabla . \overline{G}^A (r,r') \right] . \, J_s(r') \; ds' \qquad (3)$$

The laurentz gauge is :

$$\frac{j\omega}{k_i^2} \nabla . \overline{G}^A (r,r') = - \nabla' \, K_\Phi (r,r') \qquad (4)$$

The modified scalar potential is given by equation :

$$\Phi = \int_S K_\Phi (r,r')\, \nabla ' . J_s(r')\ ds' - \int_C K_\Phi (r,r')\ J_s(r') . \hat{n}\ d\ell ' \quad (5)$$

This calculation is greatly simplified in the case where the structure does not have vertical elements. There is no difficulty to consider additional terms for magnetic currents in case of dielectric surfaces. They have not been included here for sake of clarity.

III. DIPOLES INTERACTION IN THE PRESENCE OF AN INFINITE INTERFACE

The calculation of the different terms introduced in the MPIE formulation is greatly facilitated when performed in the spectral domain. In the space domain, the cylindrical coordinates are used with : $\bar{\rho} = x\hat{x} + y\hat{y}$. The corresponding variable in the spectral domain is $\bar{k}_\rho = k_x \hat{x} + k_y \hat{y}$. The field propagation is calculated using the analogy of this problem to the well known transmission line formalism using source and current sources. The transmission line equations can be written as :

$$\frac{dV^p}{dz} = -jk_{zi} Z_i^p I^p + v^p \quad (6)$$

$$\frac{dI^p}{dz} = -\frac{jk_{zi}}{Z_i^p} V^p + i^p \quad (7)$$

where v^p and i^p are the source terms.

In this expression, the superscript p stands for e in case of TM fields and for h in case of TE fields. Z is the characteristic impedance and the longitudinal propagation constant is such that

$$k_{zi} = \sqrt{k_i^2 - k_\rho^2} \quad (8)$$

where k_i is the wave number inside the medium.

In the case of two media separated by one infinite interface, the reflection coefficients for TE and TM polarized fields are given by :.

$$R^{TE} = \frac{k_{z1} - k_z}{k_{z1} + k_z} \text{ and } R^{TM} = \frac{n^2 k_z - k_{z1}}{n^2 k_z + k_{z1}}$$

The problem can easily be extended to an arbitrary number of layers [6]. The reflection coefficients become a combination of the reflection coefficients at each interface, involving the thickness of each layer. The numerical implementation is also quite straightforward.

Solutions of the transmission line equations equations have been derived by Michalski [8], [9], as the set of following equations:

$$j\omega\mu_0\ G_{zz} = \eta_0^2\ I_v^e$$

$$j\ \omega\ \mu_0\ G_{vv} = V_i^h\ ;\ G_{zu} = \frac{1}{jk_\rho}\left(I_i^h - I_i^e\right)$$

$$\text{and} \quad K_\Phi = \frac{j\ \omega\varepsilon_0}{k_\rho^2}\left(V_i^e - V_i^h\right) \quad (9)$$

IV. CALCULATION OF THE SOMMERFELD INTEGRALS

The MPIE integral terms can be written in the general form as:

$$F(\rho) = \frac{1}{2\pi} \int f(k_\rho)\ J_n(k_\rho \rho)\ k_\rho\ dk_\rho \quad (10)$$

With n equal 0 or 1. A special case of these integrals corresponds to the following, known as the Weyl identity:

$$G = \frac{\exp(-jkR)}{R} = \int_0^\infty \frac{\exp(-jk_z |z|)}{j\ k_z} J_0(k_\rho \rho) k_\rho dk_\rho$$

In the classical derivation of the problem as introduced by Sommerfeld [3], double derivatives with respect to the radial and vertical distances are introduced. This creates a difficulty when trying to implement this in a classical EFIE code with rooftop basis functions (both on wires and on surfaces). In the spectral domain, this corresponds to an increase of the Bessel function order and can be addressed more easily.

Another benefit to perform the calculation in the spectral domain is the fact that it allows isolating the different contributions, then to substract them from the integrand in order to get regular functions more suited to a numerical evaluation.

The quasi - dynamic contribution corresponds to the value of $f(k_\rho)$ when $k_\rho \rightarrow \infty$. In particular the asymptotic values of the TE and TM reflection coefficients values which appear in G_{vv} and G_{zz} are :

$$R^{TE} = \frac{k_{z1} - k_z}{k_{z1} + k_z} = 0 \quad (12)$$

And

$$R^{TM} = \frac{n^2 k_z - k_{z1}}{n^2 k_z + k_{z1}} = \frac{n^2 - 1}{n^2 + 1} = q \qquad (13)$$

and combinations of these two for the other terms.

The poles contributions are extracted by pairs [5], [1]. In the case of two layers, this only applies to the terms involving the R^{TM} coefficient. Each MPIE integral term is finally decomposed into three parts, an asymptotic value, a pole contribution and a regular part.

$$f_1(k_\rho) = f(k_\rho) - \lim_{k_\rho \to \infty} f(k_\rho) - \frac{2 k_{\rho p} R(\rho_p)}{k_\rho^2 - k_{\rho p}^2}$$

with $R(\rho_p)$ the residue of function $f(k_\rho)$ at the pole value in the k_ρ complex plane

The remaining function to be integrated after subtraction of the quasi - dynamic and of the residues parts is regular and well suited for numerical evaluation. The contribution of the quasi - dynamic contribution (QS) and of the poles is re introduced subsequently as analytical expressions.

$$F(\rho) = \frac{1}{2\pi} \int f_1(k_\rho) J_0(k_\rho \rho) k_\rho \, dk_\rho + QS\, G_0 + 2j \sum \text{Residues} \qquad (15)$$

with G_0 the free space Green function.

Two techniques may be used for numerical evaluation of the remaining integrals, either the phase stationary technique together with the steepest descent contour or the complex image technique. These two are of interest and they allow at the minimum to cross check the results obtained. For the first technique the steepest descent contour is different for each MPIE integral term. It should consider the occurrence of the poles and of the branch points.

The second technique known as complex image technique became more popular the recent years. It consists essentially in a change of the integration plane, the k_z complex plane replaces the k_ρ complex plane, and in the choice of an integration contour which ensures very fast convergence and applies to all MPIE integral terms.

The integration contour in the k_z plane is a straight line which exhibits consequently a linear relationship between the integration variable and the longitudinal wave number component. The different integrals of the remaining regular terms decrease very rapidly along this contour. They are approximated by a sum of exponential terms using the matrix pencil approximation [10]. Each one of these integrals is written as

$$f_1(k_\rho) = \sum_{1}^{N} a_i \exp(-b_i k_z) \qquad (16)$$

where N is a small number, usually below 5.
The use of Weyl identity is then made possible provided that we consider new distances defined as

$$R_i^2 = \left(\rho^2 + (z + z' - j b_i)^2 \right)^2 \qquad (17)$$

As b_i is a complex number, the result may be seen, using the Weyl identity, as a sum of complex images.

This way to proceed is illustrated on figure 1 below. The variation of the Green function terms along the contour shows a very rapid decrease. The respective contributions of the three terms depending on the radial distance between one source point and one observation point is shown on Figure 2. The weight of the different contributions strongly depends on the ground parameters.

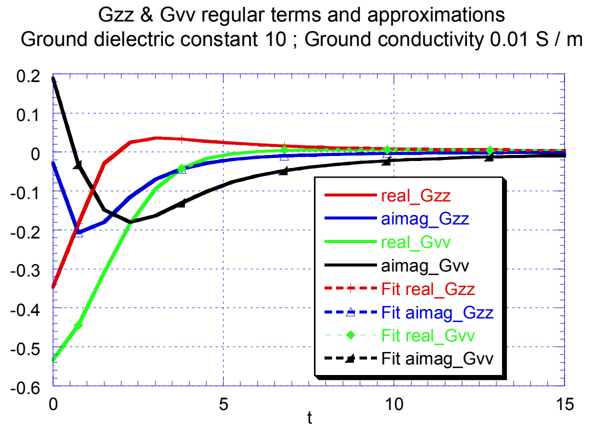

Figure 1 : Regular terms behaviour along the contour in the kz complex plane

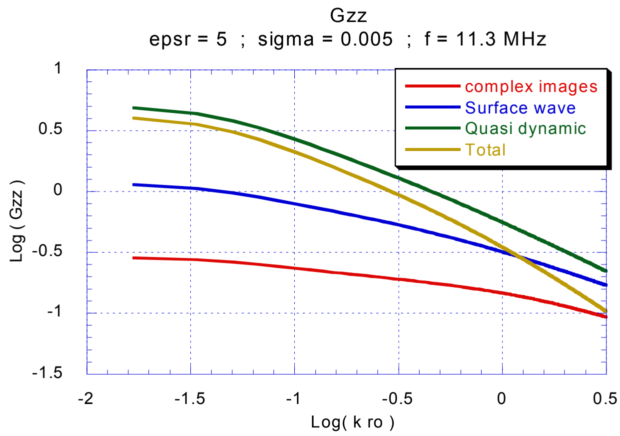

Figure 2 : Respective contributions of the Sommerfeld integrals

The extraction of the quasi dynamic contribution and of the poles contributions is still amenable in the case of an arbitrary number of layers with dipoles (either surface elements or wires) located in the different media. The different integral terms involve a combination of the different wave numbers in the different media as shown on expression below for Gzz and two media and the field radiated by a current element located in medium 1 on a current element located in medium 2.

However, in the case when one of the medium has losses, it becomes complicated to use the complex image technique to calculate the integral terms in the conducting medium for all expressions. This can only be done for some of the terms and using approximations. It is preferable in that case to return to the classical technique, finding the steepest descent contour which is a different one for each term of the Green tensor. The decomposition into the three contributions still simplifies this calculation.

V. RESULTS OBTAINED

The case considered is a typical bi conical antenna of 7 meters height placed on the ground (dielectric constant 15 and conductivity 0.01 S / m). The calculation provides the VSWR, the currents and the pattern (near field, far field and ground wave).

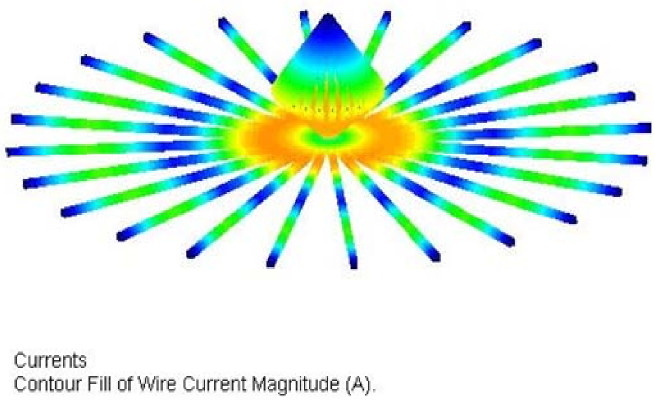

Figure 3 : Current distribution on an HF bi - conical antenna

The ground plane plays a major role on the antenna matching, allowing to decrease the VSWR to acceptable values.

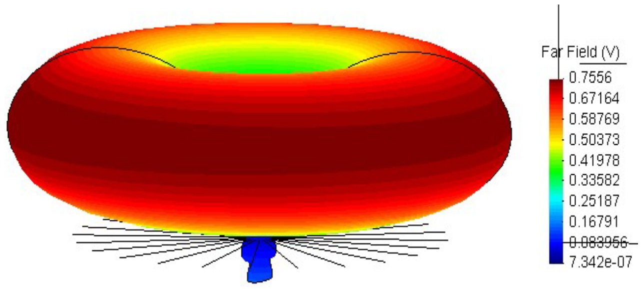

Figure 4 : Antenna far field

The sky wave far field pattern is zero in the ground plane which is usually of a small extent as compared to the wavelength.

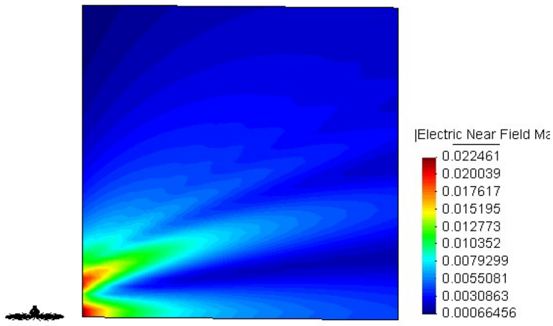

Figure 5 : Antenna near field

The near field pattern includes one additional contribution corresponding to the ground wave which exhibits a peak value in the interface plane.

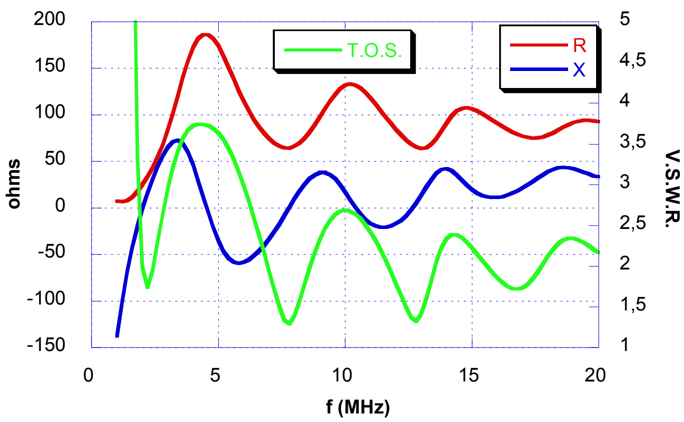

Figure 6 : Antenna input impedance and VSWR over the HF bandwidth

VI. CONCLUSION

The analysis of antennas located at the vicinity of an infinite interface was presented. It was shown how to modify the field integral equations in order to take the interface into account. The technique was presented for two media but can easily be extended to an arbitrary number of layers. Such a multiple layers structure occurs in particular in the case of microstrip antennas.

As derived by Michalski [8], the Green's problem function is a tensor. Each term of this tensor includes a combination of the TE and TM reflection coefficients. There is some benefit to split these integrals into three separate contributions corresponding to the poles contribution, to a "quasi dynamic" contribution and to a regular term which shall be calculated numerically. Two techniques may be used to evaluate the regular term: the steepest descent contour technique and the complex image technique. This last offers a number of advantages as regards in particular its implementation. The simultaneous development of these two techniques allows to verify the accuracy of the results. However, the complex image technique has some limitations and cannot be used in all cases, in particular in the case of current elements of arbitrary orientation located in lossy media.

The developments and algorithm modification have been verified by checking the results with respect to measurements on existing antennas [4]. The case of a bi – conical HF antenna has been shown. Although not presented here, the validations made on the VSWR have shown an excellent agreement.

REFERENCES

[1] Aksun.M., "A robust approach for the derivation of closed form Green's functions", IEEE MTT, vol 44, N° 5, pp. 651 – 658, May 1996

[2] Albert I. M. Romier, D. Belot, J-P Adam, P. Hamel, "Design, Manufacturing and Test of a Spline – Profile Square horn for Focal Array Applications", Antem Symposium, Toulouse, June 2012

[3] Banos A., "Dipole Radiation in the Presence of a Conducting Interface", Pergamon, 1966

[4] Béniguel Y., J-P Adam, M. Darces, F. Jangal, "Propagation par Onde de Sol en HF et VHF », Journées Guerre électronique de Toulouse, Novembre 2009

[5] Chow Y. L., J. J. Yang, D. G. Fang, G. E. Howard, "A Closed Form Spatial Green's Function for the Thick Microstrip substrate", IEEE MTT, Vol 39, N° 3, March 1991

[6] Dural G., M. Aksun, "Closed Form Green's Function for General Sources and Stratified Media", IEEE MTT, Vol 43, N° 7, July 1995

[7] http://www.ieea.fr/en/softwares/icare-mom.html

[8] Michalski K., D. Zheng, "Electromagnetic Scattering and Radiation by Surfaces of Arbitrary Shape in Layered Media, Part I: Theory", IEEE AP, Vol 38, N° 3, 1990

[9] Michalski K., J. Mosig, "Multilayered Media Green's Functions in Integral Equation Formulations", IEEE AP, Vol 45, N° 3, 1997

[10] Sarkar T., O. Pereira "Using the Matrix Pencil Method to estimate the Parameters of a Sum of Complex Exponentials", IEEE AP Magazine, Vol 37, N° 1, 1995

[11] Sommerfeld A.., "Partial Differential Equations in Physics", Academic Press, 1949