An improved Hybridization Technique of Geometrical Optics Physical Optics

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Abstract — **The methods of geometrical optics and physical optics are particularly well adapted to calculate the RCS (Radar Cross Section) of targets with dimensions very large as compared to the incident electromagnetic field wavelength. The geometrical optics (GO) technique is a very fast technique allowing to take high order interactions into account. However, it fails to calculate the scattered field on occurrence of caustics. For this reason, it is supplemented by the physical optics (PO) to obtain a full angular diagram solution. In this paper, a method to reconstitute a reflected beam issued from the impact of an incident ray on a meshed surface is presented. This method improves the precision of the GO method.**

Index terms; Numerical modeling, Physical Optics (PO), Geometrical Optics (GO), ray tracing, Radar Cross Section (RCS).

I. INTRODUCTION (HEADING 1)

The hybridization of the GO method and the PO method consists in launching a grid of rays. These rays transport an electromagnetic field, with a polarization, a phase and an amplitude. When a ray hits a surface, it generates an electric current ($\vec{J}_e$) and a magnetic current ($\vec{J}_m$) on it using the PO relation:

$$\vec{\boldsymbol{J}}_e = \widehat{\boldsymbol{n}} \times \left(\vec{\boldsymbol{H}}_i + \vec{\boldsymbol{H}}_r\right) \tag{1}$$

$$\vec{\boldsymbol{J}}_m = -\widehat{\boldsymbol{n}} \times \left(\vec{\boldsymbol{E}}_i + \vec{\boldsymbol{E}}_r\right) \tag{2}$$

Where $\vec{H}_i$ and $\vec{H}_r$ corresponds to the incident and reflected magnetic field and $\vec{\mathrm{E}}_\mathrm{i}$ and $\vec{\mathrm{E}}_\mathrm{r}$ corresponds to the electric field. A reflected ray (and eventually a transmitted ray), with a new polarization and a new direction of propagation is determined by the Descartes-Snell's law. This ray may be a part of a new grid and is launched again. When all rays have been launched, the scattered far field $\vec{E}_S(\vec{r})$ is calculated using the curents generated on the surfaces (the time convention $e^{j\omega t}$ is omitted throughout the paper):

$$\vec{E}_S(\vec{r}) = \frac{j\omega\mu}{4\pi}\frac{e^{-jkr}}{r}\hat{e}_r \times \int_S \left(\hat{e}_r \times \vec{J}_e + \frac{1}{Z_0}\vec{J}_m\right) e^{+jk\hat{e}_r.\vec{r}}dS \tag{3}$$

Where Z_0 is the free space wave impedance, $\hat{e}_r$ is the direction of observation, $\vec{r}$ the coordinates of the surface element dS, and k is the wave number. In the numerical implementation, the integral becomes a discrete summation over the surface S, meshed with triangles of indices i (surfaces dS_i).

When considering a curved surface target discretized with triangles, a drawback of the ray technique is that the scattered field is discontinuous due to the fact that the normal to the surface changes abruptly from one triangle to the next one as illustrated in Fig.1. This has been improved, dividing each incident ray into multiple reflected rays, using the normals at the triangle's vertices (see Fig.2). This method aims to get the direction of the reflected rays with a good accuracy, in a small calculation time.

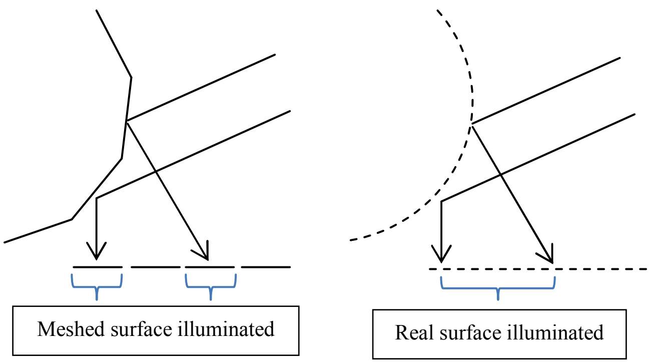


Fig.1 Comparison between meshed surface and real surface

In this paper, the reconstitution of the beam is first described, using a propagation algorithm. Then the method used to determine if a triangle is into the beam is proposed. Finally the results of a simple hybridization method, named SPOGO, is compared with the improved method presented here, named IPOGO, which also aims to reconstitute the beam and with an iterative PO method (IPO) taken as a reference.

II. ALGORITHM TO REBUILD THE REFLECTED BEAM

Usually, with the GO method, a grid of rays is launched and the surface hit by this grid is determined [1]. The method proposed here consists in determining if a surface is hit by a beam and estimate the ray direction associated to this impact. In other words, with this method, one looks from the target point of view instead of the source point of view.

Since the incident electromagnetic field is known, (a simple plane wave), the determination of the first interaction is quite easy; a test ray is just launched from a triangle which is potentially illuminated by this incident field. The direction of this ray test is opposite to the direction of the incident field. If the ray test hits another surface, it means that the triangle is in the shadow. If the triangle is illuminated, a reflected beam is defined. Its main direction is calculated from the reflection's law and eventually a transmitted beam, with main direction also calculated from the Descartes-Snell's law.

The determination of the following interactions is more complicated. First the triangle hit by the ray which direction corresponds to the main direction of the reflected (or transmitted) beam is determined. This triangle is certainly into the beam. Its neighbor's triangles are potentially into the beam as illustrated in Fig.2. A method to determine if they are certainly into the beam is presented in the third part, this method aims also to estimate the direction of the corresponding reflected ray. After it has been concluded that a triangle potentially into the beam was actually certainly into the beam, its neighbors are considered to be potentially into the beam. With this algorithm, all the triangles into the beam can be identified with a simple propagation algorithm (considering they are around the first triangle hit).

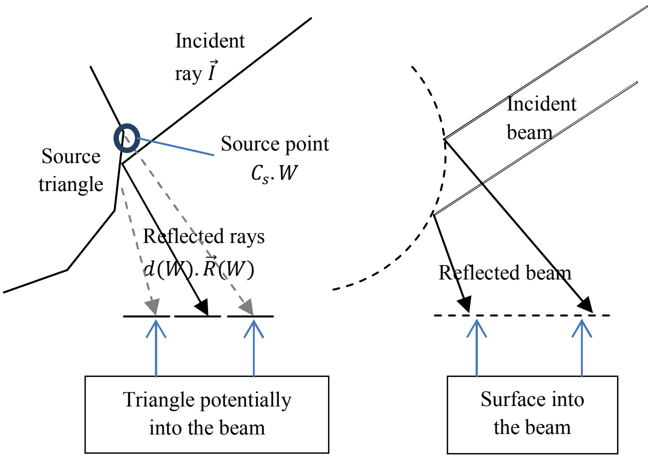


Fig.2 method to find the surface illuminated by the reflected beam

In order to illustrate and validate this propagation algorithm, a plate under an hemisphere is used. This geometry is illuminated by an incident horizontal plane wave. The wave reflected on the hemisphere is divergent and illuminates the plate as illustrated in Fig.3.

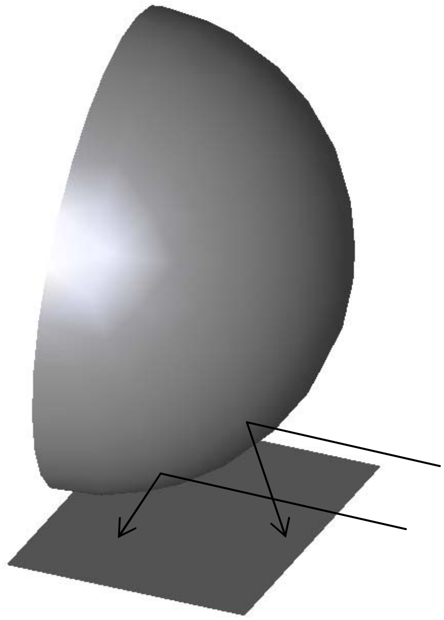


Fig.3 Real surface of the hemisphere. The reflected wave on the hemisphere is divergent on the plate

The mesh on the hemisphere is very large, while the mesh on the plate is very fine (see Fig.4). This configuration is not usual, but it aims to test the division of the incident ray into thousands of reflected rays. The reflected wave on the hemisphere will also hit a small triangle on the plate and the algorithm of propagation will recover all its neighbors triangles into the beam.

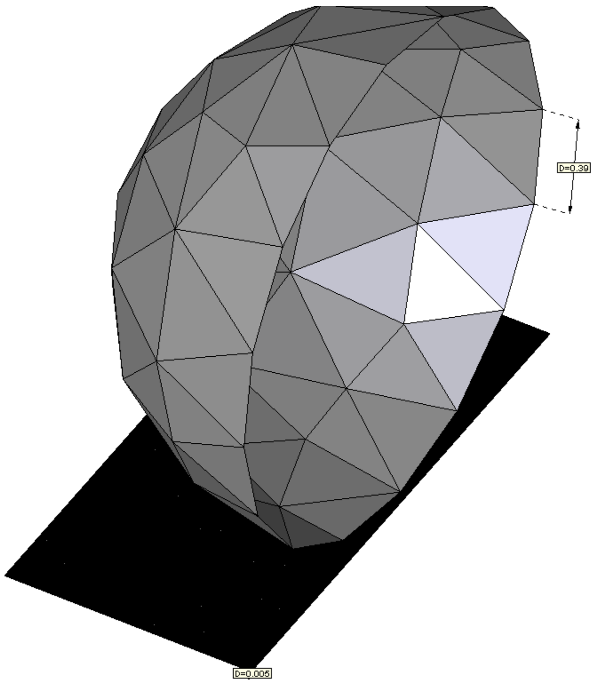


Fig.4 Meshed surface. The incident rays on the hemisphere need to be divided into multiple reflected rays on the plate.

The image on the plate generated by the reflected wave is shown in Fig.5. A color is associated to the triangles; the blue triangles are hit only one time, by the incident field. The green triangles are hit two times, the first impact is due to the incident field and the second impact is due to the reflected field on the hemisphere. The red triangles are hit 3 times. They may appear when the method to determine if a triangle is into the beam fails. In this case, the images of two triangles on the hemisphere are overlapping.

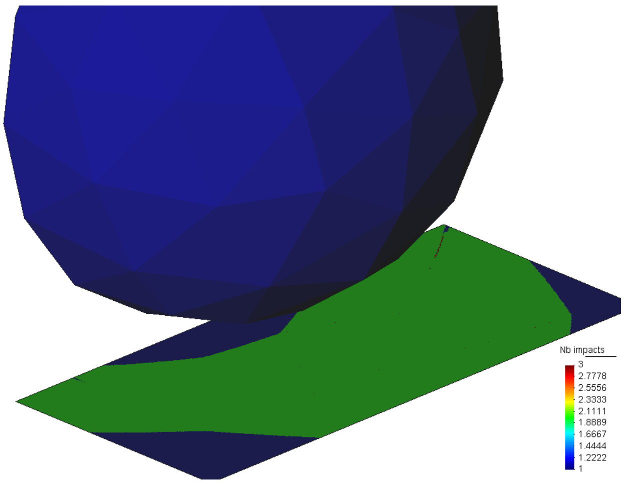


Fig.5 Image of the reflected field on the hemisphere on the plate.

The image is well retraced, even if there is some overlapping. There are also some blue triangles in the corner of the plate. This is due to the fact that the center of the beam issued from some triangles on the hemisphere never hits the plate and as a consequence, the algorithm cannot propagate on it and retrace the image. This error disappear when the size of the mesh on the hemisphere becomes smaller.

III. METHOD TO FIND THE SOURCE POINT

This method aims to determine the source point of a reflected ray which may hit a triangle potentially into a beam. This method aims also to determine if the source point is inside the source triangle and if the target triangle is certainely into the beam. A vector of unknown weight coefficient W is therfore introduced:

$$W = \begin{pmatrix} W1 \\ W2 \\ W3 \end{pmatrix} \tag{4}$$

It is assumed that the summation of the coefficient of W is equal to 1. The coordinates of the triangle's vertices (*C1*,*C2* and *C3*) for the source triangle are in the matrix:

$$C_S = \begin{pmatrix} C1x & C2x & C3x \\ C1y & C2y & C3y \\ C1z & C2z & C3z \end{pmatrix} \tag{5}$$

This aims to define the source point $C_s.W$ (c.f. Fig.2):

$$C_sW = \begin{pmatrix} C1x.W1 + C2x.W2 + C3x.W3 \\ C1y.W1 + C2y.W2 + C3y.W3 \\ C1z.W1 + C2z.W2 + C3z.W3 \end{pmatrix} \tag{6}$$

This point is in the same plane than the source triangle. The center of the triangle potentially into the beam has the following coordinates :

$$\vec{C}_P = \begin{pmatrix} C_Px \\ C_Py \\ C_Pz \end{pmatrix} \tag{7}$$

The reflected ray, which connects the source point to the center of the triangle potentially into the beam, is now defined:

$$d(W).\vec{R}(W) = \vec{C}_P - C_s.W \tag{8}$$

Where $d(W)$ is a scalar, which corresponds to the distance between the source point and the triangle potentially into the beam and $\vec{R}(W)$ is a unit vector, corresponding to the direction of the reflected ray (or transmitted ray). The normal on the triangle source vertices (*N1*, *N2* and *N3*) can be interpolated from the normals of the triangles around these vertices [2]:

$$N_S = \begin{pmatrix} N1x & N2x & N3x \\ N1y & N2y & N3y \\ N1z & N2z & N3z \end{pmatrix} \tag{9}$$

It is assumed that the direction of the incident ray at the source point is constant in the plane of the source triangle (this assumption is valid for a plane wave but it is an approximation for a non plane wave):

$$\vec{I} = \begin{pmatrix} Ix \\ Iy \\ Iz \end{pmatrix} \tag{10}$$

The direction of the normal to the surface, on the source point, aims also to introduce the relation that have to be solved in order to find W:

$$\vec{N}(W) = \frac{N_S.W}{\|N_S.W\|} = p.\frac{q.\vec{R}(W) - \vec{I}}{l(W)} \tag{11}$$

Where $l(W)$ is a scalar, $\vec{N}(W)$ is a unit vector which gives the direction of the normal and q is equal to 1 for a reflexion and corresponds to the ratio of the indices of refraction for a transmission as illustrated in Fig.6. The variable p may correct the orientation of the normal, in the case of a transmission from a dielectric to the vacuum for instance. Its value is also -1 if q is smaller than 1 and 1 in the other cases.

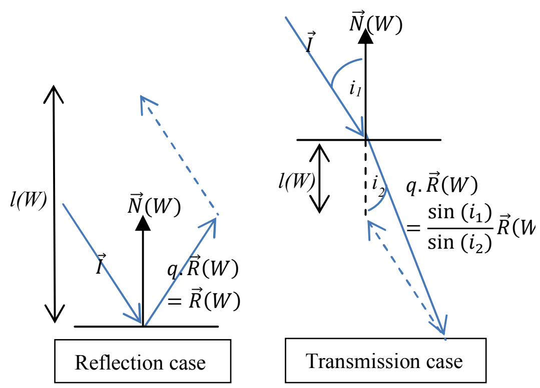


Fig.6: determination of the normal and definition of the scalar *l(W)* associated to it.

Developing (11), we get :

$$N_s.W.l(W) = \frac{p.q}{d(W)}.(\vec{C}_P - C_s.W) - p.\vec{I} \qquad (12)$$

Then W can be isolated and expressed as a function of *l(W)* and *d(W)*:

$$W = \left(N_s.l(W) + \frac{p.q.C_s}{d(W)}\right)^{-1}.\left(\frac{p.q.\vec{C}_P}{d(W)} - p.\vec{I}\right) \qquad (13)$$

Solving this equation iteratively, a weight coefficients series is created:

$$W_n = \begin{pmatrix} W1_n \\ W2_n \\ W3_n \end{pmatrix}; \qquad W_0 = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}. \qquad (14)$$

The first source point tested is also the center of the source triangle. The following terms of this series is also defined as:

$$W_{n+1} = \left(N_s.l(W_n) + \frac{p.q.C_s}{d(W_n)}\right)^{-1}.\left(\frac{p.q.\vec{C}_P}{d(W_n)} - p\vec{I}\right). \qquad (15)$$

This new coefficient is the product of a 3 by 3 matrix by a vector. Since we are only interested in the weight coefficient W and in the direction of the reflected ray $\vec{R}(W)$, the result will be the same for any origin and the problem can be translated in order to get a better conditioned number for the matrix. The following criterion is used as a convergence criterion of the W_n series:

$$\|W_{N+1} - W_N\| < \varepsilon \quad \Rightarrow \quad \lim_{n\to\infty} W_n = W_N \qquad (16)$$

Assuming that the summation of the coefficient of W_n is equal to 1, $\varepsilon = 10^{-4}$ provides a good precision for the limit of this series. A source point is also into the source triangle if the coefficients of W_n are all positives. In this case, once W_n reaches its limit, if $C_s.W_N$ is into the source triangle, the triangle hit by the reflected ray is certainly into the beam.

It is possible to transform W_{n+1} in order to have positives coefficient and keep a summation equal to 1 but it is important that the convergence criterion (16) is verified before this transformation. Once a reflected beam has been divided into multiple beams, the associated electric field needs to be normalized as in [3].

IV. COMPARISON WITH OTHER METHOD

Many test cases presented during the Workshop ISAE EM may validate the improved method (IPOGO) presented here. It will be compared with a method (SPOGO) which does not reconstitute a reflected or transmitted beam and with a reference iterative PO method (IPO).

A. Case of the placyl

The first test case is a perfect conductor plate located below a cylinder terminated on one extremity by an hemisphere Fig.7. This case named "Placyl" has been previously studied[4].

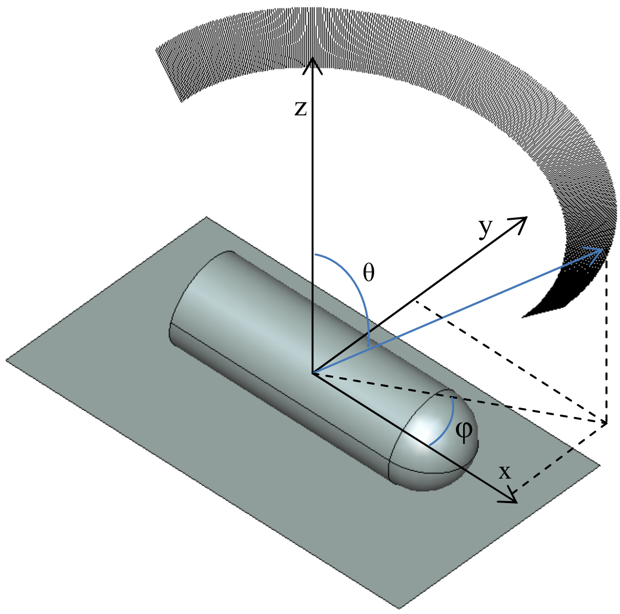
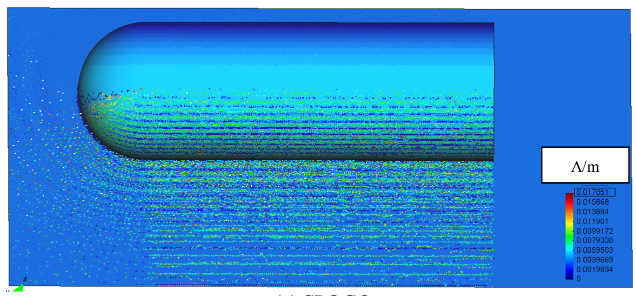
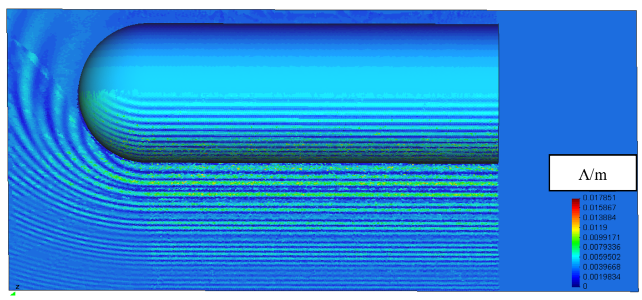


Fig.7 placyl illuminated by a plane wave, f=10GHz, θ=45°, 0°<φ<180°, the black lines correspond to the directions of observation.

The size of the plate is 1.8m x 1.2m. The radius of the cylinder and of the half sphere is 0.4m. The length of the cylinder is 1m. The mesh size is λ/5=0.6cm and the number of triangles is about 220,000. For the specific angle of observation θ=45°, φ=90° and a horizontal polarisation at the frequency f=10GHz, the magnitude of the electric current generated is plot in Fig.8. One can observe that the reconstitution of the beam provides a continuous repartition of the surface current. This may improve the accuracy of the interferences.



(a) SPOGO



(b) IPOGO

Fig.8 Magnitude of the surface current, f=10GHz, θ=45°, φ=90°, with SPOGO method (a) and with IPOGO method (b)

After 3 reflections, IPOGO provides a better results than the SPOGO as shown in Fig.9 (RCS monostatic, polarization horizontal) However, the calculation time is significantly higher for the IPOGO method (50 minutes) than for the SPOGO method which takes only 5 minutes.

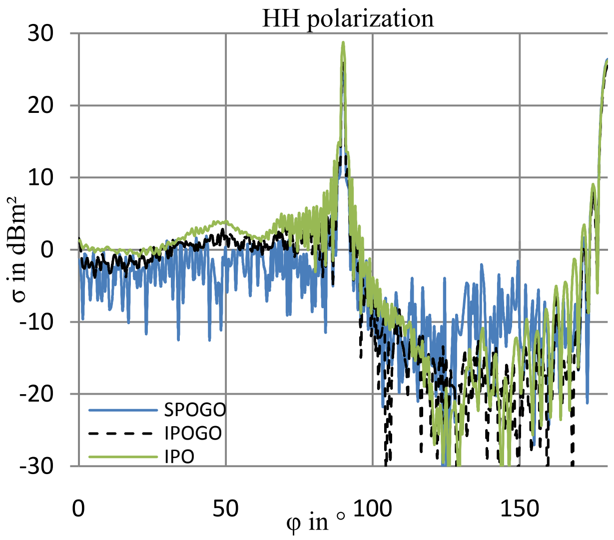


Fig.9 RCS of the placyl calculated with SPOGO and IPOGO

These results are similar to those presented in [4].

B. Case of the dielectric cube

The second test case is a dielectric cube with side length a=12cm as (see Fig.**10**) also studied by F. Weinmann [5].

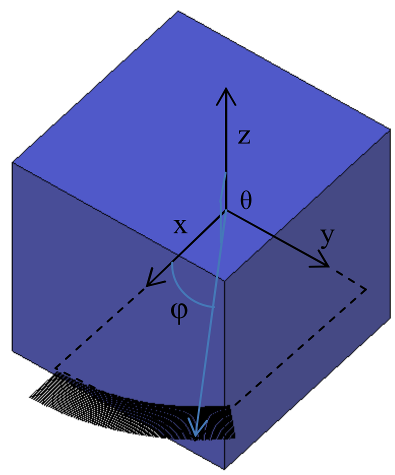


Fig.10 dielectric cube illuminated by a plane wave, f=30GHz, θ=90°, 0°<φ<45°. The dielectric constant is ε=2.7-0.01j, the black lines correspond to the directions of observation.

The mesh size is $\lambda/5$=0.2cm and the number of triangles is about 45,500. There is no curved surfaces on this geometry but the IPOGO method may improve the result, especially for grazing transmitted rays issuing from triangles on the edges. They are divided in order to illuminate the whole adjacent surface as illustrated in Fig.**11**.

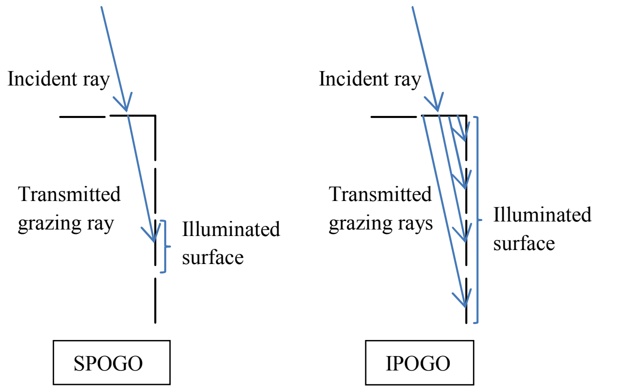
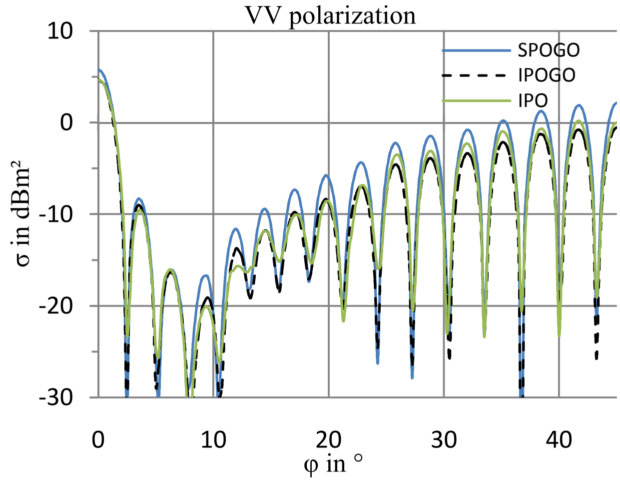
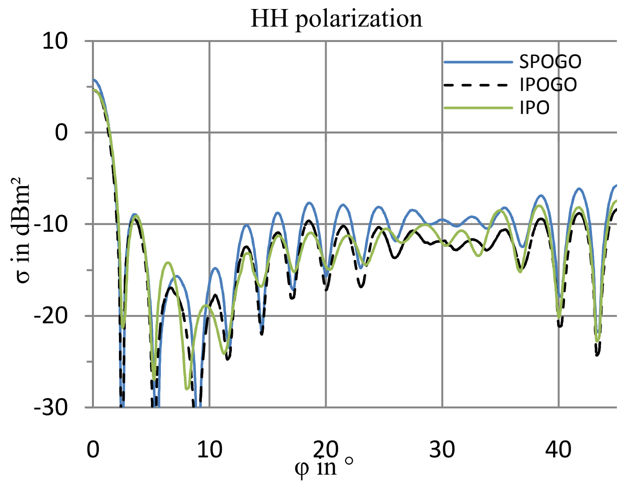


Fig.11 comparison of the surface illuminated by a transmitted grazing beam, between the SPOGO and the IPOGO method.

After 3 iterations, once again, IPOGO with a larger calculation time (30min) provides better results than SPOGO (1min) as illustrated in Fig.**12**. The results are similar to those published in [5].



(a)



(b)

Fig.12 RCS of the dielectric cube calculated with SPOGO, IPOGO and IPO for a vertical polarization (a) and horizontal polarization (b).

V. CONCLUSION

An algorithm of reconstitution of an incident beam on a meshed surface has been presented. This algorithm divides an incident ray into multiple reflected or transmitted rays. This algorithm uses a method to determine if a triangle, potentially into a beam, is or is not into it. It has been shown that this algorithm improves significantly the calculation of monostatic RCS for curved objects or planar objects. The calculation time may become critic for cases requiring to take a large number of interactions into account.

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