GISM
Global Ionospheric Scintillation Model


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- Bias
- Scintillations
- Modelling Results vs Measurements
- Scattering Function Calculation (SAR observations)
- Conclusion
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Measurement Campaigns

PRIS


http://telecom.esa.int/telecom/www/object/index.cfm?fobjectid=29210

MONITOR


MONITOR Extension

On going; start: 30 June 2014

Ionosphere Variability

TEC Map

S4 map cumulated over 24 hours

Mean Errors
(ray technique calculation)

\[ n^2 = 1 - \frac{X}{1 - \frac{Y^2 \sin \vartheta}{2(1-X)} \pm \left[ \left( \frac{Y^2 \sin^2 \vartheta}{2(1-X)} \right)^2 + Y^2 \cos^2 \vartheta \right]^{1/2}} \]

\[ X = \frac{\omega_p^2}{\omega^2} \quad Y = \frac{\omega_b}{\omega} \]

Haselgrove equations (Simplified)

\[ \frac{d x_i}{d t} = \frac{c^2 k_i}{\omega} \quad \frac{d k_i}{d t} = -\frac{\omega_p}{\omega} \frac{\partial \omega_p}{\partial x_i} \]
Bias

Range error

\[ \Delta L = \frac{\lambda^2 r_e}{2\pi} N_T \quad N_T = \int_{0}^{z} N_e \, ds \quad \Delta L = \frac{40.3 N_T}{f^2} \]

Faraday rotation

\[ \Psi = \frac{e^3}{2\varepsilon_0 c m^2 \omega^2} \int_{0}^{z} N_e \, B \cos \vartheta \, ds \]

Inputs: \( N_e \); \( B \) at any point inside ionosphere
Example of results / Solar Flux 150
HF Antenna Pattern as an input
Turbulent Ionosphere
(scintillation)
Physical Mechanism

Satellite signal

Drift velocity

Receiver level

Medium Radar Observations

Observations at Kwajalen Islands
Courtesy K. Groves, AFRL

Observations in Brazil
Courtesy E. de Paula, INPE

The vertical extent may reach hundreds of kilometers
Scintillation on Galileo Satellites
L1 vs E5a

Field Propagation Equation

\[ E(\rho, z, \omega, t) = U(\rho, z, \omega) \exp\left\{ j(\omega t - \int k(z')dz') \right\} \]

The field amplitude value \( U \) is a solution of the parabolic equation

\[ 2jk \frac{\partial U(\rho)}{\partial z} + \nabla^2 U(\rho) + k^2 \varepsilon_1(\rho) U(\rho) = 0 \]

Method of solution: phase screen technique
Solution of the parabolic equation

\[ 2 j k \frac{\partial}{\partial z} \langle U(r) \rangle + \nabla_i^2 \langle U(r) \rangle + k^2 \langle \varepsilon(r) U(r) \rangle = 0 \]

\[ 2 j k \frac{\partial}{\partial z} \langle U(r) \rangle + \nabla_i^2 \langle U(r) \rangle + j \frac{k^3}{4} A(0) \langle U(r) \rangle = 0 \]

Using the phase index autocorrelation function

\[ B(z, \rho) = \langle \varepsilon(\rho_1) \varepsilon(\rho_2) \rangle \quad A(\rho) = \int B(z, \rho) \, dz \]
Phase Screen Technique

Transmitter

Propagation

Propagation

Scattering

Scattering

Scattering

Receiver

Propagation: 1st & 3rd terms; scattering: 2nd & 3rd terms
Medium Characterization
Index Spectral Density
Medium’s Phase Spectrum

\[ \gamma_\Phi (K) = \frac{\left( \lambda r_e \right)^2 L C_s \sigma_{Ne}^2}{\left( K^2 + q_0^2 \right)^{p/2}} = \frac{C_p}{\left( K^2 + q_0^2 \right)^{p/2}} \]

3 parameters: \( \sigma_{Ne} \); \( q_0 \); \( p \)

Sample characteristics: $S_4 = 0.51$, $\sigma \phi = 0.11$
Sample characteristics: $S_4 = 0.51$, $\sigma_\phi = 0.11$
5 days RINEX files considered in the analysis

S4 > 0.2 & sigma phi < 2 (filter convergence)

2 parameters to define the spectrum: T (1 Hz value) & p
Slope spectrum vs time after sunset

slope Power spectrum

slope phase spectrum

Time after sunset

Phase variance
Time domain vs frequency domain

\[ \sigma_{\phi}^2 = 2 \int_{f_c}^{\infty} PSD(f) \, df = 2 \int_{f_c}^{\infty} T \, f^{-p} \, df = 2T \left[ \frac{f^{-p+1}}{-p+1} \right]_{f_c}^{\infty} = \frac{2T}{(p-1)f_c^{p-1}} \quad \text{(if } p > 1) \]

Slope set to 2.8
Medium Characterization
(Correlation Function)

Isotropic

1D

Anisotropic
2D Analysis: Isotropic Medium

\[ B_\Phi (\rho) = \frac{C_p}{(2\pi)^2} \iint \gamma (K) \exp(-jK \cdot \rho) \, dK \]

\[ [B_\Phi (\rho)]_{iso} = \frac{\sigma^2}{2^{(p-4)/2} \Gamma((p-2)/2)} (\rho q_0)^{(p-2)/2} K_{(p-2)/2} (\rho q_0) \]

\[ \sigma^2 = B_\Phi (0) = (\lambda r_e)^2 L L_0 \sigma_{Ne}^2 \]

1D Analysis
Isotropic Medium

\[ B_\Phi (\rho) = \frac{C_p}{2\pi} \int \gamma_\Phi (k) \exp(-jk\rho) \, dk \]

\[ [B_\Phi (\rho)]_{1D} = \frac{C_p}{2\pi} \frac{\sqrt{\pi}}{2^{(p-3)/2} \Gamma(p/2)} \quad q_0^{1-p} \quad (\rho q_0)^{(p-1)/2} \quad K_{(p-1)/2} \left( \rho q_0 \right) \]
1D vs Isotropic

Slope

\[ p \rightarrow p - 1 \]

Multiplicative factor

\[
\frac{2 \Gamma\left(\frac{p - 1}{2}\right)}{\sqrt{\pi} \Gamma\left(\frac{p}{2}\right)}
\]
Anisotropic vs Isotropic

\[ \gamma_{\Phi}(K) = \frac{\left(\lambda r_e\right)^2 L C_s \sigma_{Ne}^2 a b}{\left(q^2 + q_0^2\right)^{p/2}} \]

\[ \gamma_{\Phi}(K) = \frac{a b C_p}{\left(\left(AK_{x_\perp}^2 + BK_{x_\perp} q_{y_\perp} + CK_{y_\perp}^2\right)^2 + q_0^2\right)^{p/2}} \]

Additional geometric factor with respect to the 2D case

\[ G = \frac{ab}{\left(AC - B^2/4\right)^{1/2}} \]

a, b ellipses axes
A, B, C trigonometric terms resulting from rotations related to variable changes
\[ \Phi (\rho) = \text{FFT}^{-1} \left( \text{FFT}(u) \ast \gamma_{\Phi}(k) \right) \]

u random number with a uniform spectral density

Done at each successive layer
## Numerical Constraints

<table>
<thead>
<tr>
<th>Frequency band</th>
<th>Medium parameters$^{(1)}$</th>
<th>Phase variance$^{(2)}$</th>
<th>Aliasing$^{(3)}$</th>
<th>Propagation</th>
</tr>
</thead>
</table>
| P 450 MHz      | $L_0 = 500 \text{m}.$  
$N_e = 10^{12} \text{el/m}^3$  
RMS = 20 %             | $\sigma_\phi^2 = (\lambda r_e)^2 \Delta z L_0 \sigma_{Ne}^2$  
$\sigma_\phi = 1.41$  
($\sigma_\phi = 1.19$) | $L > \frac{z \lambda \sigma_\phi}{L_0 \sqrt{2}}$  
$z=3.10' ; \sigma_\phi = 1$ | $\Delta z < 25 \text{km}$  
$L = 2500 \text{m}$  
$\Delta x = 3.3 \text{m}$ |
| L 1.5 GHz      | $L_0 = 500 \text{m}.$  
$N_e = 10^{12} \text{el/m}^3$  
RMS = 20 %             | $\sigma_\phi^2 = 0.12$  
($\sigma_\phi = 0.36$) | $L > 85 \text{m}$  
$z=3.10' ; \sigma_\phi = 1$ | $\Delta z < 122 \text{km}$  
$L = 2500 \text{m}$  
$\Delta x = 4.88 \text{m}$  
(FFT : 1024 pts) |
| S 2.5 GHz      | $L_0 = 500 \text{m}.$  
$N_e = 10^{12} \text{el/m}^3$  
RMS = 20 %             | $\sigma_\phi^2 = 0.05$  
($\sigma_\phi = 0.22$) | $L > 51 \text{m}$  
$z=3.10' ; \sigma_\phi = 1$ | $\Delta z < 203 \text{km}$  
$L = 2500 \text{m}$  
$\Delta x = 4.88 \text{m}$  
(FFT : 1024 pts) |
Sub Models
(1 / 2)
Seasonal Dependency
(Low Latitude Scintillations)
Scintillation Events Histograms

Lima
Geographic Latitude: -12°1 / Magnetic Latitude: 0°65

Cape verde 2013 / Novatel

Cayenne
Geographic Latitude 4°8 / Magnetic Latitude: 15°2

Rwanda – 30 June 2014 – 11 July 2014
Scintillation Events / Lima 2012

S4 vs Day of year 2012 Lima

Lima 2012 / Novatel

Measurements in Malindi, Kenya
Measurements in Burkina Fasso

Koudougou (Burkina Fasso)
Geographic Latitude: 12°15 / Magnetic Latitude: -1°14

Number of Events

Sub Models
(2 / 2)

Local Time Dependency
(Low Latitude Scintillations)
One week of measurements in Guiana

Intensity standard deviation (S4)

S4 all satellites days 314 to 319 / year 2006

Local time : post sunset hours (CLS measurements, PRIS Campaign)
Checking Results

- Indices
- Inter frequency correlation
- Probability of intensity
- Fades distribution
- Loss of Lock
Medium Characterisation

Mean Effects (Sub Models)

NeQuick, Terrestrial Magnetic Field (NOAA)

Geophysical Parameters

SSN, Medium Drift Velocity

LT & Seasonal dependency

Scintillations (Fluctuating medium)

Spectrum slope (p), BubblesRMS, OuterScale (L₀)

Anisotropy ratio
Numerical Implementation

The model includes an orbit generator (GPS, Glonass, Galileo, ...)

Inputs

Medium Characterisation
Geophysical Parameters
Scenario

Intermediate calculation: LOS, Ionisation along the LOS

Outputs

Scintillation indices
Correlation Distances (Time & Space)
Scattering function

Signal at receiver level

intensity (dB) vs time (s.)
L1 / S4 = 0.68

intensity (dB) vs time
435 MHz / S4 = 1

phase (deg) vs time (s.)
L1 / sigma phi = 1.38

phase (radians) vs time
435 MHz / sigma phi = 7.
Modelling vs Measurements (Intensity)

S4 all satellites
Cayenne days 314 to 319: year 2006

Modelling vs Measurements

Modelling vs Measurements (Phase)

The phase RMS value is slightly lower than the S4 value

Modelling

Some samples exhibit high values (both measurements and modelling) due to the phase jumps

Scintillation Modelling vs Measurements

S4 Measurement in Cayenne
Latitude 4°8   Longitude 37°6   year 2011

S4 Calculated (GISM) in Cayenne
Latitude 4°8   Longitude 37°6   year 2011

Measurements

Modelling

Scintillation Index Dependency on Frequency

Galileo satellites

Inter Frequency Correlation L1 vs L2 (Modelling --> GISM)

using Yuma files

Inter Frequency Correlation
Weak scintillations vs strong scintillations

Tahiti Galileo N° 12 doy 85 / 2013
S4 (L1) = 0.21 ;  S4 (E5a) = 0.36

Tahiti Galileo N° 12 doy 85 / 2013
S4(L1) = 0.59 ;  S4(E5a) = 1.36

Inter Frequency Correlation Time Using 1 week of measurements in Tahiti

Frequency Correlation (Modelling)

Inter Frequency Correlation

Frequency correlation predicted by GISM

Weak scintillations
Strong scintillations

Loss of Lock
Loss of Lock when $\sigma_\phi >$ threshold value

The phase noise is related to the Intensity of the received signal

$$\sigma_{\phi_T}^2 = \frac{B_n}{(c / n_0) I} \left[ 1 + \frac{1}{2 \eta (c / n_0) I} \right]$$

$p(I)$ Nakagami distributed

Probability of Loss of Lock is $p(\sigma_\phi) >$ threshold value
Loss of Lock
(Measurements in Tahiti)

Loss of Locks L2 / strong scintillations

Time (s.)

Intensity (dB)

S4 (L1) = 0.44  S4 (L1) = 0.52  S4 (L1) = 0.68  S4 (L1) = 0.49  S4 (L1) = 0.55  S4 (L1) = 0.55
S4 (L2) = 0.69  S4 (L2) = 1.26  S4 (L2) = 0.68  S4 (L2) = 0.87  S4 (L2) = 0.76  S4 (L2) = 0.76
Loss of Lock
Measurements vs Modelling

Number of Loss of Lock on L2 (Measurements)

Modeling
Probability of loss of lock

3 days of analysis in Tahiti

Geographical Extent

Simultaneous Scintillation
Number of Satellites Simultaneously Corrupted by Scintillation

Probability of intensity / Modelling

GISM output

Nakagami law

\[ p(A) = \frac{2^m \cdot A^{2m-1}}{\Gamma(m)} \cdot \exp(-mA^2) \quad \text{with} \quad m = \frac{1}{S_4^2} \]
Fades Statistics

Example of equatorial scintillation in Ascension Island, in solarmax conditions (2001)

1. Real data

2. GISM simulation

Nakagami vs measurements
Radar Observations

Mutual Coherence Function
Correlation distance vs LSAR

$L_e$  Synthetic Aperture Length  $\rightarrow$  10 km

Ionosphere Effects

Free Space Resolution $\Delta x \quad \Delta y$

Ionosphere Effects

$\Delta x'$ Pulse broadening (dispersion)

$\Delta y'$ Turbulence Effect

$\Delta L$ Group Delay
Two Points - Two Frequencies
Coherence Function

\[ \Gamma (z, k_1, k_2, \rho_1, \rho_2) = \langle U_1(z, k_1, \rho_1) U_2^*(z, k_2, \rho_2) \rangle \]

Using the parabolic equation

\[
\left[ \frac{\partial}{\partial z} - \frac{j}{2} \frac{k_d}{k_0^2} \nabla_d^2 + \frac{k_p^4}{8 k_0^2} \left[ \frac{k_d^2}{k_0^2} A_\xi (0) + D_\xi (\rho) \right] \right] \Gamma (k_d, z, \rho) = 0
\]

The structure function \( D_\Phi(z, \rho) = 2 [B_\Phi(0) - B_\Phi(\rho)] \) is quadratic with respect to the distance.

Same process than previously: propagation 1st & 3rd terms; Diffraction: 2nd & 3rd terms.
Two Points - Two Frequencies
Coherence Function

\[ \Gamma(z, k_1, k_2, \rho_1, \rho_2) = \langle U_1(z, k_1, \rho_1) U_2^*(z, k_2, \rho_2) \rangle \]
Scattering $\rightarrow$ 2 constants

\[ B = \frac{\sigma^2_{\Phi}}{2 \omega^2} \]

\[ S = \sigma^2_{\Phi} (L_1)^2 \frac{\log (L_0 / \ell_1)}{6 L_0^2} \]

Propagation $\rightarrow$ 1 constant

\[ P = \frac{1}{2 c k^2} \left( \frac{1}{L_1 + L_2} - \frac{1}{L_1} \right) \]

* Nickisch, RS 92, Knepp & Nickisch, RS, 2010

Analytical solution

\[ \Gamma (\tau, K_x, z) = \frac{z}{2 \sqrt{B S}} \exp \left( -\frac{z^2 K_x^2}{4 S} \right) \exp \left( -\frac{(\tau + z^2 K_x^2 P)^2}{4 B} \right) \]
K = 0 \rightarrow \tau_1 = 2A \sqrt{B}

K = K\text{Max} \rightarrow \tau_2 = 4SP\,A

Spread factor

Q = \frac{\tau_2}{\tau_1} = \frac{2\sqrt{A}}{\sqrt{B}} SP
Spreading Extent

10 MHz
Very large value of Q

100 MHz
Large value of Q

435 MHz
Q around 1


The algorithm can easily be generalized

Different statistical properties may be assigned to the different layers

Numerical FFT (1D) shall be performed to get the coherence function
Ambiguity Function

\[ \chi (r, r_0) = \sum_n \int g_n(t, r_n) f_n^*(t, r_{0n}) \, dt \]

\[ g_n \text{ is the received signal and } f_n \text{ is the matched filter} \]

\[ \chi_n (r_n, r_{0n}) = \frac{1}{(4 \pi r_n)^2} \exp \left( j \Phi_0 \right) \int \exp \left( - j (\omega - \omega_0) \Phi_1 - (\omega - \omega_0)^2 \Phi_2 \right) \, d\omega \]

Value of Coherent field received

\[ \langle \chi (r, r_0) \rangle = \frac{\exp \left( - \sigma_\phi^2 \right)}{(4 \pi r_0)^2} \int_{-L_e/2}^{L_e/2} \exp \left( 2 j k_0 \left( r_0 + \frac{\rho^2}{2 r_0} \right) \right) \, d\rho = \frac{\exp \left( - \sigma_\phi^2 \right)}{(4 \pi r_0)^2} \sin \left( k_0 \rho L_e / r_0 \right) \]

An attenuation factor on the coherent component is included
Coherent Length

It is given by function

\[ \Gamma (\omega_d, \rho, r) = \sqrt{\frac{D}{S}} \exp \left( -B \omega_d^2 - D \left( \frac{\rho}{r_0} \right)^2 \right) \]
Positioning Errors
Modelling

S4 measured for each tracked satellite

\[ \sigma_t \] is calculated taking the thermal noise as the main contribution

Range error calculated assuming a gaussian distribution

GPS constellation simulated with a yuma file
GPS Positioning Errors

Unregistered HyperCam 2

Positioning errors from Measurements in Brazil in 2001
Conclusion

- Reasonable agreement between modelling (GISM) and measurements

- Positioning errors due to scintillations may reach values up to 50 meters

- The azimuthal resolution of a SAR may be significantly decreased

- All results will be updated taking measurements campaign data into account